Mechanism Design: Single Agent, Discrete Types

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Introduction to Mechanism Design

- Mechanism Design Theory pertains to the design of incentive schemes by a Principal for one or more Agents endowed with superior payoff-relevant information
- Examples:
  - Monopolist selling to customers privately informed about their tastes
  - Employer designing performance incentives for workers privately informed about their productivity
  - Government designing income tax schemes for citizens privately informed about their abilities or needs
  - Regulator designing price regulations for a public utility privately informed about its technology
  - Auctioneer designing an auction to sell an indivisible object to potential buyers privately informed about their valuations
  - Bargaining intermediary/arbitrator designing a mechanism for sale of an object by a seller to a buyer, both privately informed about their valuations

Introduction, contd.

- In all these contexts:
  - Principal and Agents have different objectives
  - P has the power to design the mechanism
  - Agents have better information
- Focus is on implications of the informational asymmetry for resource allocation, especially (in-)efficiency, and distribution of welfare
- Diverse applications in IO, public finance, macro, labor, development

Introduction, contd.

- Distinguish from one other kind of Principal-Agent problem, involving *moral hazard (MH)*
- MH: where some actions taken by Agents are not observable to the Principal (*hidden information* versus *hidden actions*) — will be considered in last two weeks of the course
- Some contexts may involve both hidden action and hidden information (e.g. productivity may be privately known, *and* depend on unobservable effort of the agent) — we shall not cover these

Monopolist Problem, Two Types of Consumers

- P is a profit maximizing monopolist selling a good to consumers with heterogeneous valuations of the good
- Good is produced at constant unit cost c, no capacity constraint
- P's payoff from selling $q \geq 0$ units of the good (or quality of the good) in exchange for payment T received from a customer is $i \equiv T - cq$
- Customer's payoff is $U \equiv \theta V(q) - T$, where θ (*type*) is privately known by the customer, where V is strictly increasing, strictly concave and $V(0) = 0$
- Simplest case: two possible types $(0 <)\underline{\theta} < \bar{\theta}$

Monopolist Problem, Two Types of Consumers, contd.

- P makes a take-it-or-leave-it offer to each customer, not knowing the type of the latter
- What P knows (from past experience or market surveys) is the distribution of types in the population: fraction $\underline{p}$ are of type $\underline{\theta}$, and $\bar{p} \equiv 1 - \underline{p}$ are of type $\bar{\theta}$
- Each customer is free to not buy the good — i.e., always has the option to select $q = T = 0$ and receive 0 utility (*outside option*)

Perfect Information Benchmark

- What would happen in this world if P were perfectly informed about each customer's type (say, upon acquiring personalized data from Facebook)?
- Could fine-tune the offer to each customer type (*perfect price discrimination*)
- Offer $(T(\theta), q(\theta)$ to a type θ customer to maximize $T - cq$ subject to $\theta V(q) - T \geq 0$
- Solve in two steps:
  - Given any q select the highest possible payment that the customer will accept: $T^*(q) = \theta V(q)$
  - Choose q to maximize $T^*(q) - cq$
- **Solution:** $\theta V'(q(\theta)) = c, \ T(\theta) = \theta V(q(\theta))$
- Allocation is efficient, but P appropriates all the surplus

Formulation as a Bayesian game of incomplete information

- Return to the case of asymmetric information, where P does not know each customer's type
- Refer to the representative consumer as the Agent, who is privately informed about her type
- Sequence of moves:
  - A learns realization of $\theta \in \{\underline{\theta}, \bar{\theta}\}$ (Nature's move)
  - P offers a 'contract'/'mechanism' which is a game
  - A agrees to play the game, or walk away and receive 0
  - If A agrees to play, the game is played

Meaning of ‘Contract’ or ‘Mechanism’

- The ‘contract’/game specifies sequence of moves and strategy spaces at each stage: e.g.,
  - (one-shot) posted-‘price’ mechanism (function $T(q)$)
  - a dynamic process of negotiation (offer (T, q)—counteroffer (T', q')—-counter-counteroffer $(T", q")$,...)
- Feasible set of mechanisms is very large and complicated
- Fortunately, however, can restrict attention to a very simple set of mechanisms: posted-prices; nothing to be gained by entering into protracted bargaining (the **Revelation Principle**)
- *Key underlying assumption:* P is able to commit to the mechanism, cannot renegotiate it in the middle

Revelation Principle (RP)

- RP states that P can confine attention to (one-shot) *revelation mechanisms*, where P asks A what her type is and decides on the allocation based on this report
- A revelation mechanism is a function $(T(\theta), q(\theta))$ specifying an allocation or exchange corresponding to a report $\theta \in \{\underline{\theta}, \bar{\theta}\}$ submitted by A of her type to P
- There is no way that P can figure out whether A is reporting truthfully
- RP says attention can further be restricted to mechanisms in which A is provided an incentive to report truthfully (*incentive compatible (IC)*) and to agree to participate (meets participation (P) constraints whereby A's payoff is at least 0)

Argument Underlying Revelation Principle

- Consider any (possible dynamic) game designed by P; and (possibly some refinement) of Bayesian Nash equilibrium
- Let $(T(\theta), q(\theta))$ be the allocation resulting in this equilibrium, when A is of true type θ
- Type θ has the option of mimicking the strategy played by any other type θ' and realizing the allocation $(T(\theta'), q(\theta'))$
- So type θ (weakly) prefers the allocation $(T(\theta), q(\theta))$ to $(T(\theta'), q(\theta'))$
- Hence no type of A has an incentive to lie when asked to report her type in the revelation mechanism (RM) $(T(.), q(.))$ — RM is incentive compatible
- Allocation that results in RM is the same as in the Bayesian Nash equilibrium of the original game, so must satisfy participation constraints

Feasible Revelation Mechanisms

- Denote $(\underline{T}, \underline{q}) \equiv (T(\underline{\theta}), q(\underline{\theta}))$ and $(\bar{T}, \bar{q}) \equiv (T(\bar{\theta}), q(\bar{\theta}))$, allocations for the two types respectively
- IC constraint:

$$\underline{\theta} V(\underline{q}) - \underline{T} \geq \underline{\theta} V(\bar{q}) - \bar{T}, \bar{\theta} V(\bar{q}) - \bar{T} \geq \bar{\theta} V(\underline{q}) - \underline{T} \tag{1}$$

- P constraint:

$$\underline{\theta} V(\underline{q}) - \underline{T} \geq 0, \bar{\theta} V(\bar{q}) - \bar{T} \geq 0 \tag{2}$$

- P's profit:

$$\underline{p}[\underline{T} - c\underline{q}] + \bar{p}[\bar{T} - c\bar{q}] \tag{3}$$

- P's problem now reduces to selecting the pair of allocations $(\underline{T}, \underline{q}), (\bar{T}, \bar{q})$ to maximize (**??**) subject to (**??**) and (**??**)

Steps in Solving the Problem

- Call HIC the IC of the high type $\bar{\theta}$ not wanting to pretend to be the low type, and HPC the PC of the high type
- Analogously: LIC and LPC for IC and PC for the low type
- We have four constraints, but can prune some of them
- First show that PC of the high type can be dropped, as high type always obtains higher payoff from any allocation compared to the low type

Lemma

UIC and LPC implies UPC.

Proof: $\bar{\theta}V(\bar{q}) - \bar{T} \geq \bar{\theta}V(\underline{q}) - \underline{T} \geq \underline{\theta}V(\underline{q}) - \underline{T} \geq 0.$ □

Steps in Solving the Problem, contd.

- Next show that high type must consume at least as much as the low type

Lemma

UIC and LIC imply $\bar{q} \geq \underline{q}$.

Proof: Restate UIC and LIC as:

$$\bar{\theta}[V(\bar{q}) - V(\underline{q})] \geq \bar{T} - \underline{T} \geq \underline{\theta}[V(\bar{q}) - V(\underline{q})] \quad (4)$$

which implies

$$[\bar{\theta} - \underline{\theta}][V(\bar{q}) - V(\underline{q})] \geq 0 \quad (5)$$

□

Two Step Approach

- **Step 1:** For any given pair of quantities $\bar{q} \geq \underline{q}$, find optimal payments $\bar{T}^*, \underline{T}^*$ and thus the corresponding expected revenue $R(\bar{q}, \underline{q}) \equiv \underline{p}\,\underline{T}^* + \bar{p}\bar{T}^*$

- **Step 2:** Choose optimal quantities to maximize $R(\bar{q}, \underline{q}) - c[\bar{p}\bar{q} + \underline{pq}]$

Step 1 Problem and Solution

Given $\bar{q} \geq \underline{q}$, select $\bar{T}, \underline{T}$ to maximize $\underline{p}\underline{T} + \bar{p}\bar{T}$ subject to:

$$\bar{\theta}[V(\bar{q}) - V(\underline{q})] \geq \bar{T} - \underline{T} \geq \underline{\theta}[V(\bar{q}) - V(\underline{q})] \qquad (UIC, LIC)$$

$$\underline{\theta}V(\underline{q}) - \underline{T} \geq 0 \qquad (LPC)$$

Optimal to set $\bar{T} - \underline{T} = \bar{\theta}[V(\bar{q}) - V(\underline{q})]$ and then $\underline{T} = \underline{\theta}V(\underline{q})$

Solution:

$\underline{T} = \underline{\theta}V(\underline{q})$, $\bar{T} = \bar{\theta}[V(\bar{q}) - V(\underline{q})] + \underline{\theta}V(\underline{q}) = \bar{\theta}V(\bar{q}) - [\bar{\theta} - \underline{\theta}]V(\underline{q})$

$\longrightarrow$ low type will not retain any surplus, high type will obtain a surplus or *informational rent* $[\bar{\theta} - \underline{\theta}]V(\underline{q})$, which is positive iff $\underline{q} > 0$

Step 2 Problem

- Given $\bar{q} \geq \underline{q}$, corresponding maximal revenue is

$$\underline{p}\underline{\theta}V(\underline{q}) + \bar{p}\{\bar{\theta}V(\bar{q}) - [\bar{\theta} - \underline{\theta}]V(\underline{q})\}$$
$$= \bar{p}\bar{\theta}V(\bar{q}) + \{\underline{p}\underline{\theta} - \bar{p}[\bar{\theta} - \underline{\theta}]\}V(\underline{q})$$

- and corresponding profit is

$$\bar{p}[\bar{\theta}V(\bar{q}) - c\bar{q}] + \underline{p}\{[\underline{\theta} - \frac{\bar{p}}{\underline{p}}(\bar{\theta} - \underline{\theta})]V(\underline{q}) - c\underline{q}\}$$

- **Solution:** $\bar{\theta}V'(\bar{q}) = c, [\underline{\theta} - \frac{\bar{p}}{\underline{p}}(\bar{\theta} - \underline{\theta})]V'(\underline{q}) = c$
- Quantity for high type is efficient, but for low type is inefficiently low

Properties of the Second-Best Solution

- Quantity/quality is inefficiently low for the low type (think of airline food or space in economy class)
- Because of the externality exerted on the contract between the high type and P (improving quality in economy class would cause the high types to switch into economy)
- High type retains some surplus, but low type customers do not
- Asymmetric information transfers some surplus from P to high type customers, at the cost of creating a 'distortion' in the contract with low type customers
- A central theme of this literature: *asymmetric information creates a trade-off between rent extraction and efficiency*