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EC 703b, PROBLEM SET NO. 3

1. There are two agents. Agent 1 is a seller and agent 2 a buyer of an indivisible good. θ_1 is 1's cost of producing a good and θ_2 is the buyer's valuation for this good. The θ_i's are independent and distributed uniformly on $[0, 1]$. An allocation is a pair (q, y) where q is the quantity of the good sold by the seller to the buyer (where this must be either 0 or 1) and y is the payment made by the buyer to the seller. The utility functions are

$$u_1(q, y, \theta_1) = y - q\theta_1$$

and

$$u_2(q, y, \theta_2) = \theta_2 q - y.$$

Consider the following 'double auction' mechanism. The players simultaneously choose bids, b_1 and b_2 (think of b_1 as the price proposed by the seller and b_2 as the price proposed by the buyer). If $b_1 > b_2$, the demands are incompatible, and $q = y = 0$. If $b_1 \leq b_2$, then $q = 1$ and the price is determined by splitting the difference — that is, $y = (b_1 + b_2)/2$.

(a) Find a Bayesian Nash equilibrium of this game. (Hint: Look for one where $b_1^*(\theta_1) = \alpha_1 + \beta_1\theta_1$ and $b_2^*(\theta_2) = \alpha_2 + \beta_2\theta_2$ for some constants α_i and β_i.) Is the resulting allocation *ex post* Pareto efficient?

(b) Consider the direct revelation mechanism which selects the allocation that results in this equilibrium i.e., if the seller and buyer report $\hat{\theta}_1, \hat{\theta}_2$ respectively, select the allocation that results in the equilibrium in (a) when their true types are $\hat{\theta}_1, \hat{\theta}_2$. Show that truthful reporting is a Bayesian Nash Equilibrium of this mechanism.

(c) How does this problem relate to the Myerson-Satterthwaite Theorem?

2. Suppose we have two agents, 1 and 2, and each owns one apple. The value to i per apple is θ_i where $\theta_i \sim U[0, 1]$ and θ_1 and θ_2 are independent. Consider the following mechanism.

The agents simultaneously choose bids. If i's bid is strictly larger than j's, then i pays his bid to j and consumes both apples. If the bids are tied, each agent just consumes his own apple and no payments are made.

(a) Show that there is a Bayesian Nash equilibrium where i's strategy is $\sigma_i(\theta_i) = \alpha\theta_i$ and find α.

(b) Compute the interim payoffs. Does each agent always prefer to participate in the mechanism (assuming that he can simply consume his own apple if he doesn't participate)? Is the outcome ex post efficient?

(c) How does this problem relate to the Myerson–Satterthwaite Theorem?