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EC 703b, PROBLEM SET NO. 2

1. There are n districts $i = 1, \ldots, n$ where n is an odd number. Each district i elects a representative in Congress. Congress is considering a bill to pass a foreign policy reform. Passage of the bill is denoted by $d = 1$, otherwise the status quo policy continues (denoted $d = 0$). Representative (and every citizen) of district i obtains incremental utility (valued in dollars) of θ_i from the reform, compared with the status quo. θ_i could be any positive or negative real number. The payoff of a district i citizen (and representative) is $\theta_i d - T_i$, where $T_i \geq 0$ is a financial contribution from district i's budget to the national Treasury for a foreign aid fund for disaster relief in Third World countries (that the citizens do not care about). The political mechanism prescribes a set of strategies for each representative, which determines an outcome of the foreign policy bill and financial contributions from each district. An *allocation* is an outcome $d(\theta_1, \ldots, \theta_n), \{T_i(\theta_1, \ldots, \theta_n)\}_{i=1,\ldots,n}$ of the political process for each possible realization of $(\theta_1, \ldots, \theta_n)$. In what follows, ignore the (zero probability) event that $\sum_i \theta_i = 0$, or $\theta_i = 0$ for any i.

(a) Suppose $(\theta_1, \ldots, \theta_n)$ is publicly known. What is the set of Pareto efficient allocations? Derive the associated (efficient) foreign policy outcome $d^F(\theta_1, \ldots, \theta_n)$.

(b) Now consider the situation where the realization of θ_i is known privately by this representative, and unknown to representatives of other districts. Suppose the foreign policy bill is decided on the basis of majority voting, where each representative is asked to vote yes or no for the bill ($v_i \in \{1, 0\}$), and the bill passes (i.e., $d = 1$) if and only if a majority of representatives vote in favor ($N_1 > N_0$, where N_i denotes the number of votes i). There are no required financial contributions in this mechanism ($T_i \equiv 0, \forall i$). Is there a dominant strategy equilibrium in this voting game? If so, describe the equilibrium strategies, and the resulting allocation. Is the allocation Pareto efficient?

(c) Consider a variant of the voting game, where representatives are allowed to vote for or against the bill, and in addition 'bid' by offering a financial contribution. Promised contributions by district i takes the form of a nonnegative integer k_i such that $T_i = k_i\epsilon$ for a fixed (small) number ϵ. Hence each representative's strategy is (v_i, k_i) with $v_i \in \{1, 0\}$, and k_i a non-negative integer. The political process counts the aggregate contributions C_1, C_0 of those voting for and against the bill respectively, and selects the side that promises a larger contribution (i.e., $d = 1$ if $C_1 > C_0$, and 0 otherwise). Does this game have a dominant strategy equilibrium? If so, describe the equilibrium strategies, and the resulting allocation. Is the allocation Pareto efficient?

(d) Now suppose Congress implements the VCG mechanism, where district i representative reports a value of b_i from passage of the bill, which results in foreign policy decision $d^F(b_1, \ldots, b_n)$, and contributions T_i satisfying: (a) $T_i = -B_{-i} \equiv \sum_{j \neq i} b_j$ if $B_{-i} \leq 0, b_i + B_{-i} > 0$; (b) $T_i = B_{-i}$ if $B_{-i} \geq 0, b_i + B_{-i} < 0$, and (c) $T_i = 0$ in all other situations. Does this game have a dominant strategy equilibrium? If so, describe any such equilibrium and the resulting allocation. Is it Pareto efficient?