

## EC 703b, PROBLEM SET NO. 1

1. A monopolist wishes to sell a good produced at constant unit cost  $c \in (0, 1)$  to a large population of consumers with heterogeneous preferences: a consumer of type  $\theta$  has a payoff  $\theta \log(q + 1) - t$  for consuming  $q \geq 0$  units of the good and paying  $t$  dollars for it. The monopolist cannot identify the type of any given consumer. Each customer has an outside option of 0.

(a) There are two types of customers, with  $\theta$  equal to 2, 1, and fraction  $\beta_i > 0, i = 1, 2$  of the population is of type  $i$ . Solve for the optimal contract for each type, when (i) the monopolist can identify the type of each customer, and (ii) when the monopolist cannot identify the type of each customer.

(b) Now consider the case where  $\theta$  is distributed uniformly on the interval  $[0, 1]$ .

- (i) If  $q(\theta)$  denotes the quantity sold to type  $\theta$ , find a condition on this function  $q(\cdot)$  that ensures that it is IC (incentive compatible, i.e., there exists some pricing rule  $t(q)$  for which  $q(\theta)$  is the optimal purchase of type  $\theta$ ).
- (ii) For any such IC  $q(\cdot)$ , what is the associated set of payments (i.e.,  $t(\theta)$ ) that customers (of type  $\theta$ ) make to the monopolist?
- (iii) Obtain an expression for total profit of the monopolist as a function only of the selling strategy  $q(\cdot)$  and payoff of consumer of type 0.
- (iv) Calculate the optimal selling strategy  $q^*(\theta)$ , and find the corresponding schedule of payments  $t^*(\theta)$ .
- (v) Find the payment rule  $t(q)$  that implements this outcome, i.e., where a consumer of type  $\theta$  selects  $q^*(\theta)$  to maximize  $\theta \log[1 + q] - t(q)$  and  $t^*(\theta) = t(q^*(\theta))$ . Does the optimal nonlinear pricing rule involve unit price discounts or premia for high  $q$  purchases?