Ec 703b: Additional Problems

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1. The owner of a firm hires a manager whose productivity is θ. The manager knows his productivity, but the owner does not. The owner only knows that $\theta \in \{\theta_L, \theta_H\}$, where $\theta_L < \theta_H$ and where p is the probability that $\theta = \theta_H$. The owner must hire this manager (so the individual rationality constraint must be satisfied for both types). The manager's outside opportunity is worth 0. The manager's utility function if he receives wage w, puts in effort e, and has productivity θ is $w - (e/\theta)$. The owner's utility in the same circumstances are $2\sqrt{e} - w$.

(a) If the owner can observe θ, what is the optimal contract? What are the owner's expected profits?

(b) What is the optimal contract if the owner cannot observe θ?

(c) Now suppose that there is a probability α that the owner observes the manager's type after the manager chooses a contract. More specifically, now the timing is as follows: First, the owner offers a contract. Then the manager accepts or rejects. If he accepts, the manager chooses an effort level and profits are realized. Also, with probability $\alpha \in (0, 1)$, independently of what has happened in the past, the owner observes the manager's type. Finally, the manager is paid according to the contract. What is the optimal contract?

2. Suppose there is a buyer B and seller S of an indivisible object, with payoffs $d\theta_B - t$ and $t - d\theta_S$, where $d \in \{0, 1\}$ denotes whether trade takes place, and t is a transfer from B to S. B and S are privately informed about θ_B and θ_S respectively, which are drawn independently from $[0, 1]$ according to distribution functions F_B, F_S that are common knowledge among them. Let $d^*(\theta_B, \theta_S)$ denote the ex post efficient trading rule.

(a) Derive $d^*(\theta_B, \theta_S)$ the ex post efficient trading rule.

(b) Show that any payment rule $t(\theta_B, \theta_S)$ implements $d^*(\theta_B, \theta_S)$ in dominant strategies if and only if there exist real valued functions $B(\theta_S), S(\theta_B)$ such that

$$t(\theta_B, \theta_S) = \theta_S d^*(\theta_B, \theta_S) + B(\theta_S) = \theta_B d^*(\theta_B, \theta_S) + S(\theta_B) \tag{1}$$

(c) Show that $B(\theta_B) = S(\theta_S) = k$ for some constant k.

(d) Use the above results to show there cannot exist any payment rule which implements the efficient trading rule in dominant strategies.

(e) Does there exist a payment rule which implements the efficient trading rule as a Bayesian equilibrium? Can you find such a payment rule?

3. Suppose we modify the private value auction model discussed in class by assuming there are two objects for sale, not just one. These objects are identical. Bidder i's value for one object is θ_i and has no value for a second object. The θ_i's are independent and identically distributed. Assume there are at least three bidders.

Consider the following auction: Each bidder submits a bid. The two highest bidders receive one of the objects. They each pay the third highest bid. Show that it is a dominant strategy for i to bid his value.

4. Consider a principal who earns profits from two different tasks undertaken by the agent. His profits from task 1, denoted π_1, depend randomly on the level of effort undertaken by the agent in task 1. More specifically, if the agent puts high effort e_H into task 1, then

$$\pi_1 = \begin{cases} A & \text{with probability } 1/2 \\ 0 & \text{otherwise.} \end{cases}$$

If the agent puts low effort (e_L) into task 1, then $\pi_1 = 0$ with probability 1. The principal's profits from task 2, π_2, are a *nonstochastic* function of the agent's effort on task 2. More specifically, if the agent puts high effort into task 2, then $\pi_2 = B$, while if he puts in low effort, $\pi_2 = C$ where $B > C$. The agent can only put high effort into *at most* one task. That is, the only possibilities are low effort on both tasks, e_H on task 1 and e_L on task 2, or e_H on task 2 and e_L on task 1. The agent's payoff if he is paid w and puts high effort into one of the tasks is $\sqrt{w} - g$, while his payoff if he puts low effort into both tasks and is paid w is $\sqrt{w}$. The agent's payoff if he does not work for this principal is 0. The principal is risk neutral.

Finally, assume that $A/2 > B - C > g^2$. In other words, the expected marginal value of high effort on task 1 is greater than the marginal value of high effort on task 2 and both are greater than the utility cost of high effort.

Suppose that the principal observes both π_1 and π_2 but not the agent's effort choice. Find the cost–minimizing contracts for inducing each possible effort choice. Under what conditions is the effort choice changed from the first–best?