Ec 703b: Problem Set 5

Mookherjee; Spring 2019

Due: Wednesday May 1

1. Consider the moral hazard model covered in class, but now the Principal is risk-averse, with a payoff function $y(\pi - w)$ which is strictly increasing and strictly concave. The Agent A is also risk-averse with a payoff function $v(w) - g(e)$ where v and g are both strictly increasing and v is strictly concave. All other assumptions are the same, i.e., (LLN) holds

(a) Set up the first-best problem where e is contractible. Characterize the optimal contract using Kuhn-Tucker conditions. Show that P and A must share risk: A's payment $w_i^*(e^F)$ is strictly increasing in i (where $\pi_i > \pi_{i-1}$ and e^F denotes the first best effort).

(b) Now consider the second-best problem where e is not contractible. As in class, assume there are two possible effort levels $e_L < e_H$, and $v(\underline{w})$ is low enough that (LL) does not bind. Suppose the second-best effort is e_H. Characterize the optimal contract using Kuhn-Tucker conditions. Does the (IC) constraint bind? If it always does, provide a proof; otherwise find a condition for (IC) to bind.

(c) Show that in the second-best contract, w_i is always strictly increasing in i if MLRP holds.

(d) Does the second-best contract always satisfy the 'Informativeness Principle'?

2. Return now to the case where P is risk-neutral. Consider the moral hazard problem with three possible effort levels $E = \{e_1, e_2, e_3\}$ and two possible profit levels, $\pi_H = 10$ and $\pi_L = 0$. Let the probability of π_H given e be:

$$f(\pi_H \mid e) = \begin{cases} 2/3, & \text{if } e = e_1 \\ 1/2, & \text{if } e = e_2 \\ 1/3, & \text{if } e = e_3 \end{cases}$$

The agent's cost function for effort is

$$g(e) = \begin{cases} 5/3, & \text{if } e = e_1 \\ 8/5, & \text{if } e = e_2 \\ 4/3, & \text{if } e = e_3 \end{cases}$$

The agent's utility of wealth function is $v(w) = \sqrt{w}$. His reservation utility $\bar{u}$ is zero.

(a) Show that if effort is not observable, then it is not possible to induce effort level e_2. (Notational suggestion: Let $v_1 = v(w(\pi_H))$ and $v_2 = v(w(\pi_L))$.) For what values of $g(e_2)$ can the principal induce effort level e_2?

(b) What is the optimal contract when effort is not observable?