Ec 703b: Problem Set 4

Spring 2019

Due: Tuesday April 23

1. Suppose we have I bidders with independent private values distributed uniformly on the interval $[0, 1]$. Suppose these bidders are risk averse — more specifically, if bidder i's valuation is θ_i and he gets the good at price p_i, his payoff is $(\theta_i - p_i)^\alpha$ where $\alpha \in (0, 1)$. (Restrict attention to bids which never exceed θ_i, so $\theta_i - p_i \geq 0$ and the payoff is always well-defined.)

(a) Show that there is an equilibrium in the first price auction where i bids $a\theta_i$ for some constant a. Find the constant.

(b) Find an equilibrium for the second price auction.

(c) Do the two auctions yield the same expected revenues? If not, which yields more?

2. We have I bidders in an independent private values auction. The value of the object to bidder i is $\theta_i \sim U[0, 1]$. Suppose the seller uses an all–pay auction. That is, each bidder i puts in a bid $b_i \in [0, \infty)$.

If $b_i > \max_{j \neq i} b_j$, then i receives the object. If we have a tie, we randomize uniformly over the bidders who bid the most to determine which gets the object. But in all cases, *all* bidders pay their bids.

Show that there is an equilibrium where bidder i's strategy is $\sigma_i(\theta_i) = \alpha\theta_i^\beta$ for some α and β and find α and β. Calculate the seller's expected revenue in this equilibrium. Without calculating the revenue for a first price auction, what can you say about how the revenue in the all–pay auction compares to revenue in the first price auction?

3. You want to sell an indivisible object which you personally do not value. There are two potential bidders, with independent private values. Bidder 1's value is drawn uniformly over $[0, 1]$, while bidder 2's value is drawn uniformly over $[\alpha, 1 - \alpha]$, where $\alpha \in (\frac{1}{3}, \frac{1}{2})$. Derive the optimal expected revenue maximizing sealed-bid auction in which bidders have incentives to report their true valuations. Describe the way that the bidding rules in the optimal auction 'favor' one bidder over another, and provide some intuition for this result.