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EC703

PROBLEM SET 7

1. **(Continuation of Problem 2 from Problem Set 6.)** Consider the exchange economy described in that problem: two physical goods $l = 1, 2$, two consumers that share the same von-Neumann Morgenstern utility function $\log c_1 + \log c_2$, where c_i denotes consumption of good i; two states of nature A and B. Both consumers have endowment w of the second good in either state. In state A the first consumer has endowment $\frac{w}{2}$ of the first good, while the second consumer has endowment $\frac{3w}{2}$ of this good. In state B their endowments of the first good get reversed. Both consumers assign equal probability to the two states.

(c) Suppose the consumers trade in Arrow securities (corresponding respectively to the two states) at $t = 0$ before the state of nature is revealed, and subsequently trade in commodities after the state is revealed. Given a set of state-contingent *ex post* commodity spot prices anticipated *ex ante* by each consumer (p_{1j}, p_{2j}), derive the optimal asset demand z^{iA}, z^{iB} of each consumer i as a function of date 0 security prices.

(d) Given a set of anticipated *ex post* commodity spot prices, calculate security prices that clear the securities market. Finally, derive a Radner equilibrium for this economy. Is the resulting allocation the unique Radner equilibrium allocation?

2. An exchange economy has L physical goods $l = 1, \ldots, L$, S states of nature $s = 1, \ldots, S$, and I households $i = 1, \ldots, I$. Household i has endowment vector ω_{is} in state s, beliefs π_{is} over states of nature, and von-Neumann Morgenstern utility function $u_{is}(x_{is})$ which is differentiable and concave.

(a) Provide a clear and brief definition of an *ex ante* Pareto optimal allocation for this economy.

(b) Derive first order conditions that characterize an *ex ante* Pareto optimal allocation.

(c) Now suppose there is a complete set of Arrow securities which are traded before the state of nature is revealed. Provide a clear and concise definition of a Radner equilibrium for this economy.

(d) Formulate household i's problem of selecting an optimal asset portfolio, using the *ex post* indirect utility function $V_{is}(p_s, W_{is})$ that corresponds to the direct utility function u_{is}, where p_s denotes the spot commodity prices, and W_{is} the financial wealth of the household in state s resulting from the chosen portfolio in state s.

(e) Use the first order conditions from the portfolio choice problem of households to obtain a direct proof that the Radner equilibrium of the economy is *ex ante* Pareto optimal.

3. An exchange economy has two dates $t = 0, 1$ and two states of nature $s = 1, 2$ which will be revealed at date 1. Unlike the model in class, agents in this economy do have endowments, consume and trade in goods at $t = 0$. Use $s = 0$ to denote the date-event pair corresponding to date 0. There is one physical commodity, and two consumers $i = 1, 2$ whose endowments ω_{is} are as follows: $\omega_{10} = 2, \omega_{11} = 4, \omega_{12} = 3, \omega_{20} = 4, \omega_{21} = 2, \omega_{22} = 3$. Both share the von-Neumann-Moregenstern utility $\log c_0 + \log c_1$, where c_t denotes date t consumption. Consumer 1 believes $s = 1$ will occur with probability $\frac{3}{4}$, while consumer 2 believes $s = 1$ will occur with probability $\frac{1}{4}$. At date 0, there is a spot commodity (i.e., for delivery at $s = 0$) market, besides two assets $k = 1, 2$ whose date-1 returns r_{sk} are given by $r_{11} = 1, r_{12} = 2, r_{21} = 0, r_{22} = 1$. So consumers divide their date 0 wealth between consumption at $t = 0$ and purchasing assets that yield returns at $t = 1$. At date 1, agents realize their asset returns and trade in spot commodity markets after the state is revealed.

(a) Derive the entire set of *ex ante* Pareto optimal allocations in this economy. Are these allocations *ex post* Pareto optimal as well?

(b) Describe carefully the optimization problem defining the optimal asset demands of the two consumers at date 0 (You need not derive the asset demand functions: show the objective function and the budget constraints.)

(c) What can you say about existence and Pareto optimality of Radner equilibria in this economy?