

Dilip Mookherjee

EC703

PROBLEM SET 6

1. An exchange economy has two dates $t = 0, 1$, one consumption good $L = 1$, I households, and S states. Households consume nonnegative quantities of the good at **both** dates. Household i has endowment $\omega_{0i} > 0$ of the good at date 0, and ω_{si} at date 1 in state s . Let $W_s \equiv \sum_{i=1}^I \omega_{is}$ denote the aggregate endowment of the consumption good in this economy in state s at $t = 1$.

The von Neumann Morgenstern utility function of household i is given by $u_i(x_{0i}) + u_i(x_{si})$, where x_{0i} and x_{si} respectively denote consumption at date 0 and at date 1 in state s respectively. The function u_i is strictly increasing, strictly concave, twice continuously differentiable, and $u_i'(0) = \infty$. All households believe that state s will arise with the same probability $\pi_s > 0$.

(i) Set up the problem of finding the set of *ex ante* Pareto efficient consumption allocation in this economy (in terms of a constrained optimization problem).

(ii) Show that any *ex ante* efficient allocation has the property that date 1 consumption of all agents move ‘together’, i.e., for any i there exists a strictly increasing function f_i such that $x_{si} = f_i(W_s)$, all s .

(iii) Show that Pareto optimal risk sharing takes a linear form: $f_i(W_s) = a_i + b_i W_s$ if either (a) each household has constant absolute risk aversion, or (b) each household has constant relative risk aversion which is the same across all households.

2. Consider an exchange economy with two physical goods $l = 1, 2$, and two consumers that share the same von-Neumann Morgenstern utility function $\log c_1 + \log c_2$, where c_i denotes consumption of good i . There are two states of nature A and B . Both consumers

have endowment w of the second good in either state. In state A the first consumer has endowment $\frac{w}{2}$ of the first good, while the second consumer has endowment $\frac{3w}{2}$ of this good. In state B their endowments of the first good get reversed. Both consumers assign equal probability to the two states.

(a) Derive the entire set of *ex ante* Pareto optimal allocations in this economy.

(b) Derive the set of competitive equilibrium allocations in the corresponding Arrow-Debreu economy with a complete set of contingent commodity markets that open before the state of nature is revealed. (Here derive the excess demand function for each household and then solve for equilibrium prices by the condition that aggregate excess demand for each contingent commodity vanishes.) Verify directly that each of these equilibria belongs to the set of *ex ante* Pareto optimal allocations you obtain in (a) above.