Ec 703

Dilip Mookherjee

PROBLEM SET NO. 4

1. (a) Consider an exchange economy where every household has strictly convex, strictly monotone preferences, and $Z_i(p)$ is the excess demand function for household i defined for all strictly positive price vectors p. State conditions on these functions that guarantee existence of a competitive equilibrium.

(b) Now add to this economy a production sector in which any firm's technology is described by a compact and strictly convex production set which permits inactivity (i.e., includes the null vector). Verify that this economy has a well-defined aggregate excess demand function which also satisfies the same conditions as in (a) above, and hence must have a competitive equilibrium.

2. Consider an exchange economy with L commodities and I households, where household i has consumption set R^L_+, a nonnegative endowment vector, and a utility function $U_i(x_{i1}, x_{i2}, \ldots) = \min\{\frac{x_{i1}}{\lambda_{i1}}, \frac{x_{i2}}{\lambda_{i2}}, \ldots\}$, with $\lambda_{il} > 0$ for all i, l. Provide a complete proof that this economy has a competitive equilibrium.

3. Apply the Index Theorem to show that competitive equilibrium must be unique in an economy where every household i has a utility function of the form $U_i = \phi_i(x_{1i}, x_{2i}, \ldots, x_{L-1,i}) + x_{Li}$, where ϕ_i is a strictly monotone and strictly concave function.