PROBLEM SET 2: Ec 703, Spring 2008

due January 28, 2008

1. This is a continuation of the last problem on Problem Set 1. In the economy defined in that problem:

(a) Consider any interior Pareto optimal allocation (i.e., where consumption, leisure and time worked are positive for all persons). Construct the prices and lump-sum transfers for each person that will decentralize this, as described by the Second Welfare Theorem. Then check that the allocation is indeed a price equilibrium at the prices and transfers that you have constructed.

(b) Suppose the social planner's equity goals are described by a utilitarian welfare function $\sum_i U_i$, i.e., where every person's utility U_i is assigned a welfare weight of unity. Calculate the allocation which maximizes this measure of social welfare.

(c) Now use your answer to (a) above, and calculate the lump sum transfers and prices that will decentralize the welfare optimal allocation described in (b) above.

(d) Describe in economic terms the formula for lump sum transfers you have obtained in (c). For instance how do they vary with a person's ability? What does this tell you about the difficulties of implementing these transfers if each person knows his own ability and the planner does not?

2. An economy has two goods C (a chemical product) and L (laundry services). The C good is produced by a C-firm, and the L good by an L-firm. The C-firm emits an effluent which pollutes the local air quality and lowers the productivity of the L-firm. Both firms use a single factor: labor services, supplied by local households. The production function of the C-firm is $y_c = \alpha.W_c$, where α is a positive parameter, and W_c is the amount of work (labor services) procured by the C-firm. The production function of the L-firm is $y_L = \beta.\gamma(y_c).W_L$, where W_L is the amount of work procured, β is a positive parameter, and $\gamma(.)$ is a strictly decreasing, twice-differentiable positive-valued function that represents the negative externality imposed by the C-firm on the L-firm.

There is a single (representative) household in the economy, which consumes the two goods and supplies labor services to the two firms, and has a utility function $C^\delta L^\eta (1-W)^\psi$, where δ, η, ψ are positive parameters; $C, L \geq 0$ denote consumption of the two goods, and $W \in [0,1]$ the aggregate time worked. The household has no endowment of either good, and a unit endowment of time. The household owns all the shares in the two firms.

(a) Define a Pareto optimal allocation for this economy. Obtain first order conditions that this allocation must satisfy.

(b) Define a competitive equilibrium allocation (with the assumption that the L-firm takes the production decisions of the C-firm as given when making its own production decision, and both firms take prices as given). Obtain first-order conditions satisfied at this equilibrium.

Compare them with the first order conditions for Pareto optimality (from (a) above), and comment on whether the First Welfare Theorem applies.

(c) Now suppose the government mandates that in order to produce the C-good, the C-firm has to purchase production permits from the L-firm, at the rate of one permit per unit of the C-good produced. Each firm will take the price of the C-permits as given, and maximize profits (so the value of the permits traded will be a cost for the C-firm and a revenue for the L-firm). Extend the notion of a competitive equilibrium to this economy, and call it a Lindahl equilibrium. Obtain first-order conditions satisfied at this equilibrium. Compare them with the first order conditions for Pareto optimality (from (a) above).

(d) Finally, evaluate whether the Second Welfare Theorem applies to Lindahl equilibria: if yes provide any required assumptions and a proof; otherwise show that it does not apply and explain the reason.