PROBLEM SET 1: Ec 703, Spring 2008

due January 22, 2008

1. Show that if there are L goods in the economy and consumers have locally non-satiated preferences, and all firms and consumers take prices as given and respectively maximize profits and utility subject to a budget constraint, then Walras's Law holds: if at some price vector p, $L-1$ markets clear ($\sum_i(x^*_{il} - \omega_{il}) = \sum_j y^*_{jl}, l = 1, \ldots, L-1$), then the last ($L$-th) market must also clear ($\sum_i(x^*_{iL} - \omega_{iL}) = \sum_j y^*_{jL}$).

2. Consider a pure exchange economy with consumers $i = 1\ldots, I$, where consumer i has endowment $\omega_i \in \Re^L_+$ and locally non-satiated preferences on consumption set $X^i = \Re^L_+$. A *coalition* C is a nonempty subset of the set of consumers $\{1, \ldots, I\}$. It *blocks* an allocation $\{x^*_i\}_{i=1,\ldots}$ if there exists $x_j \in \Re^L_+, j \in C$ such that every $j \in C$ strictly prefers x_j to x^*_j, and $\sum_{j\in C} x_j = \sum_{j\in C} \omega_j$. Prove that a competitive equilibrium allocation cannot be blocked by any coalition.

3. There are I persons in an economy indexed $1 = 1, \ldots, I$. There are two goods: corn and labor services. Corn is produced in farms with a constant returns to scale technology, where one unit of labor service produces w units of corn. Person i consumes $c_i \geq 0$ units of corn, and spends $t_i \in [0, 1]$ fraction of the day working: he is then able to deliver $n_i.t_i$ units of labor services (where $n_i \geq 0$ represents the ability of i, a fixed parameter for each person). Different persons have differing abilities. Everyone has the same utility function $u(c_i) + d(1 - t_i)$ where u, d are both strictly increasing, strictly concave twice-differentiable functions satisfying Inada conditions ($U'(0) = d'(0) = \infty, U'(\infty) = d'(1) = 0$).

(a) Describe the set of allocations in this economy.

(b) Define a Pareto-optimal allocation for this economy. Set up a maximization problem whose solution generates the set of Pareto optimal allocations. Derive the first-order-conditions that characterize (i.e., are necessary and sufficient) for Pareto optimality. Explain how there can be multiple Pareto optimal allocations in the economy.

(c) Define a competitive equilibrium for this economy. Derive first-order conditions that characterize a competitive equilibrium. Use these and the characterization in (2) above to establish that every competitive equilibrium in Pareto optimal.