An economy has two goods: boat rentals and money. There is a fixed stock of boats in the economy owned by a number of sellers who derive no personal benefit from a boat ride, and care only about the amount of money they can earn from renting out their boats. The sellers sell boat rentals at a price determined by a competitive market.

There are a large number of identical purchasers of boat rentals, none of whom own a boat, and whose utility from a boat ride depends on the weather. The state of the weather can take one of four values \( s = +2, +1, -1, -2 \). In state \( s \), the utility of a renter is \( \beta^s \log t - p \cdot t \) where \( \beta \) is a positive number, \( t \) is the amount of time that the boat is rented for, and \( p \) is the price paid per unit time of the rental. Each renter assigns equal prior probability to the four possible states. Let \( T \) denote the fixed supply of boat rental time in this economy per consumer.

(a) Suppose initially that boats are rented before any information about the weather is available. Describe the market equilibrium in this situation, and compute the level of \((ex ante)\) welfare achieved by each consumer.

(b) Now suppose a weather forecast is available which publicly announces whether the sign of \( s \) will be positive or negative. Represent this signal as an information partition of the set of states of the weather, and describe the posterior beliefs that consumers will have consequent on any particular forecast.

(c) Suppose the price of boat rentals is fixed at \( p \). Compute the optimal demand for boat rentals for any given consumer, following any given forecast. Then compute the private value of the weather forecasting service to each consumer (while continuing to assume that the price is fixed at \( p \)).
(d) Now suppose the price is determined in a competitive market. Compute the equilibrium price following any given forecast.

(e) Finally, compute the (social) value of the forecasting service to each consumer, and each seller (assuming that sellers are risk neutral).