Ec 703 Microeconomic Theory

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1. [**2+3+10=15 marks**] An exchange economy has two dates $t = 0, 1$, one physical good ($L = 1$), I households, and S states of nature. Households hold endowments of goods and consume only at $t = 1$. In state s (known at $t = 1$ but not at $t = 0$), i consumes $x_{is} \geq 0$, has endowment $\omega_{is} > 0$, and a von-Neumann Morgenstern utility function $u_{is}(x_{is})$ which is continuous, strictly increasing and strictly concave. At $t = 0$, household i believes state s will arise with probability $\pi_{is}(> 0$ for all $i, s)$. There are K assets in this economy, with asset k generating return r_{sk} of the commodity in state s, where $r_{sk} > 0$ for all s. Household i has an initial endowment $\bar{z}_{ik} > 0$ of asset k at $t = 0$. Households trade in assets at $t = 0$, but are subject to a short sales constraint: $z_{ik} \geq -B$, for all i, k, where z_{ik} is i's holding of asset k at $t = 1$ and B is a positive number.

(i) Define the expected utility of household i at $t = 0$ as a function of its asset portfolio.

(ii) Define a competitive equilibrium in the asset market at $t = 0$.

(iii) Show that such an equilibrium must exist. What role, if any, does the short sales constraint play in your argument? (You can invoke Propositions that have been proven in class or in problem sets, there is no need to prove them again. But make sure to check the validity of conditions required by any Proposition you invoke).

2. [**10 marks**] Consider an exchange economy with three goods, and one type of consumer with Cobb-Douglas preferences and strictly positive endowment of each good. Show by direct computation that the index of every (strictly positive) price vector in this economy must be +1.

3. [**5*5 = 25 marks**] Consider an exchange economy with $L(\geq 2)$ goods, I households, S states of the world $s = 1, \ldots S$ at $t = 1$, and trading in K financial assets at $t = 0$, where asset k pays off $r_{sk} > 0$ units of good 1 in state s. These assets trade at $t = 0$, while spot commodity markets open at $t = 1$ after s is revealed. All households share the same belief at $t = 0$ that state s will arise with probability $\pi_s > 0$. Household i has a state-independent von-Neumann Morgenstern utility function $u_i(x_{i1}, x_{i2}, \ldots, x_{iL})$ which is strictly increasing, strictly concave, twice continuously differentiable, with the marginal utility of any good l equal to ∞ if good l consumption equals zero.

In contrast to a Radner equilibrium where households correctly anticipate at $t = 0$ the commodity prices that will prevail at $t = 1$ in any given state, consider instead the Hicksian notion of a *temporary equilibrium* where household i expects an exogenously given spot price $\bar{p}_{ils}$ for good l to prevail in state s. These price expectations may differ across households and turn out to not be actually realized.

(a) Formulate the expected utility at $t = 0$ of household i as a function of its asset portfolio (and given the prices it anticipates at $t = 1$).

(b) Provide a complete definition of a temporary equilibrium allocation and prices (for assets at $t = 0$ and date 1 spot commodity prices), given the price expectations of the agents. Explain how the definition differs from a Radner equilibrium.

(c) Define an *ex ante* Pareto optimal allocation in this economy, and obtain first-order conditions that characterize such an allocation.

(d) Define an *ex post* Pareto optimal allocation in this economy, and obtain first-order conditions that characterize such an allocation. Explain how these differ from those that characterize an *ex ante* Pareto optimal allocation.

(e) What can you say about the *ex ante* or *ex post* Pareto optimality of a temporary equilibrium allocation?