PROBLEM SET 11, SOLUTIONS

1. (a) Type $i = h, l$ has expected utility: $EU_i = p_iU(Y - d + r) + (1 - p_i)U(Y - m)$, and generates expected profit to a firm: $\Pi_i = (1 - p_i)m - p_i r$. (b) The slope of the indifference curve for type $i$ is $\frac{dr}{dm} = \frac{1 - p_i}{p_i} \frac{U'(Y - m)}{U'(Y - d + r)}$, so indifference curves are upward sloping, convex to the $m-$ axis, and flatter for the $h$ type. (c) The zero profit line is $r = \frac{1 - p_i}{p_i} m$, so is an upward sloping straight line passing through the origin, with a lower slope for the high risk type. (d) Full insurance requires $r + m = d$, so is a downward sloping straight line with a slope of $-1$, passing through $(0, d)$ and $(d, 0)$. (e) Max $p_iU(Y_d + r) + (1 - p_i)U(Y - m)$, subject to $(1 - p_i)m = p_i r$. The solution is full insurance $r + m = d$, at actuarially fair odds $r = \frac{1 - p_i}{p_i} m$, so the solution is $m = p_i d$ and $r = (1 - p_i)d$. (f) It is not incentive compatible, since the $h$ types pay more in premiums and receive a smaller payout in the event of an accident. So each high risk customer would masquerade as a low risk customer.

(e) A separating Spencian equilibrium would be a pair of contracts $(m_i, r_i), i = h, l$ such that:

\[
p_h U(Y - d + r_h) + (1 - p_h)U(Y - m_h) \geq \max\{\bar{U}_h, p_h U(Y - d + r_l) + (1 - p_h)U(Y - m_l)\} \tag{1}
\]
\[
p_l U(Y - d + r_l) + (1 - p_l)U(Y - m_l) \geq \max\{\bar{U}_l, p_l U(Y - d + r_l) + (1 - p_l)U(Y - m_h)\} \tag{2}
\]
where $\bar{U}_i \equiv p_iU(Y - d) + p_iU(Y)$, and the zero profit conditions

\[
p_i r_i = (1 - p_i) m_i, i = h, l \tag{3}
\]

One separating Spencian equilibrium is where the high risk type gets his first-best full insurance contract, and the low risk contract gets the contract which leaves the high risk customer indifferent between the two contracts, and firms break-even on the low risk types. The low risk types are then under-insured. We can reduce the extent of insurance for the low risk types slightly and still preserve the incentive compatibility constraints. This allocation
will be Pareto dominated by the previous one since it involves greater underinsurance for the low risk customers.

(h) A pooling Spencian equilibrium is a single contract \((m, r)\) offered to both types which breaks even:

\[
\{\lambda(1 - p_h) + (1 - \lambda)(1 - p_l)\}m = \{\lambda p_h + (1 - \lambda)p_l\}r
\]

and in which each type attains at least his autarkic utility:

\[
p_i U(Y - d + r) + (1 - p_i) U(Y - m) \geq \bar{U}_i, i = h, l
\]

Any policy lying on the zero profit line for pooled contracts (i.e., satisfying (4)) which does not involve ‘excessive’ insurance (i.e., so that high risk types are at least as well off as in autarky) is a pooled Spencian equilibrium. It includes the full insurance contract satisfying (4).

2. A PBE is \(p^L, p^H, \beta(.), \sigma(.)\) where \(\beta(p)\) is the belief of all firms after observing a premium offer of \(p\) that the customer is low risk, and \(\sigma(p)\) is the subsequent probability of accepting the offer.

Denote the payoff of type \(i = H, L\) customer from offering premium \(p\) in this equilibrium:

\[
W^i(p|\sigma(.)) = \sigma(p) U(Y - p) + [1 - \sigma(p)] [\pi^L U(Y - L) + (1 - \pi^i) U(Y)]
\]

and \(P(p)\) the profit of a firm from accepting a premium offer of \(p\):

\[
P(p) = p - L[\beta(p)\pi^L + (1 - \beta(p))\pi^H]
\]

A PBE must satisfy the following constraints:

(i) For each type \(i\), \(p^i\) maximizes \(W^i(p|\sigma(.))\), and \(W^i(p^i|\sigma(p^i)) \geq W^i \equiv \pi^i U(Y - L) + (1 - \pi^i) U(Y)\)

(ii) Firms accept all profitable offers \((\sigma(p) = 1 \text{ if } P(p) > 0)\), reject all unprofitable ones \((\sigma(p) = 0 \text{ if } P(p) < 0)\), and \(\sigma(p)\) is anything between 0 and 1 if \(P(p) = 0\).
(iii) $\beta(p)$ is obtained from offer strategies $p^i$ on the equilibrium path, i.e., $\beta(p^L) = 1, \beta(p^H) = 0$ if $p^L \neq p^H$, and $\beta(p^*) = 1 - \alpha$ if $p^L = p^H = p^*$.

(b) There cannot be any separating PBE where the offers of both types are accepted, because then the type offering to pay the higher premium would do better to offer the lower premium instead.

(c) A pooling PBE $p^*$ must satisfy the following: $\beta(p^*) = 1 - \alpha$, $P(p^*) = p^* - L[\alpha \pi^H + (1 - \alpha) \pi^L] \geq 0$ and $U(Y - p^*) \geq \bar{W}$ if $\sigma(p^*) > 0$. Focusing attention on pooled equilibrium where the offer $p^*$ is accepted with positive probability, the above two participation constraints impose the following upper and lower bounds on $p^*$.

Let $\bar{p}$ denote $L[\alpha \pi^H + (1 - \alpha) \pi^L]$, and let $\tilde{p}$ denote the solution to $U(Y - \tilde{p}) = \pi^L U(Y - L) + (1 - \pi^L) U(Y)$. Then $\bar{p} \leq p^* \leq \tilde{p}$ must be satisfied by every pooled PBE outcome. In addition, if $\bar{p}^H$ denotes $L\pi^H$, then firms will accept any $p > \bar{p}^H$ because they will make profits at any such $p$, irrespective of beliefs. Then no such $p$ can be a pooled PBE outcome, as both types would be better off offering a slightly lower $p$. Hence a pooled PBE must additionally satisfy $p^* \leq \bar{p}^H$. We therefore obtain the following necessary condition:

$$\bar{p} \leq p^* \leq \min\{\tilde{p}, \bar{p}^H\} \quad (*)$$

Any $p^*$ satisfying (*) can be the outcome of a PBE if combined with the following strategies and beliefs of firms: $\sigma(p) = 1, \beta(p) = 1 - \alpha$ for all $p \geq p^*$, $\sigma(p) = 0, \beta(p) = 0$ for all $p < p^*$. So (*) is a necessary and sufficient condition for $p^*$ to constitute the outcome of a pooled PBE.