1. An insurance market has a large number of customers that belong to two risk categories: high and low, with probabilities of an accident being \( p_h \) and \( p_l \) respectively, where \( 1 > p_h > p_l > 0 \). A given fraction \( \lambda \in (0, 1) \) of the customers are the high-risk types. There is a single consumption good. All customers have the same income \( Y \) in the absence of an accident, and suffer a damage of \( d \in (0, Y) \) in the event of an accident. They share a common von Neumann-Morgenstern utility function \( u \) which is strictly increasing, strictly concave and differentiable. There are a large number of risk-neutral insurance firms competing to offer insurance policies to these customers. An insurance policy is described by a premium \( m \) paid by the customer to the insurance company when there is no accident, and a net payout \( r \) from the insurance company to the customer in the event of an accident.

(a) Write down the expected utility of an insurance customer of either type, and the expected profit of the insurance company, as a function of the insurance policy \((r, m)\).

(b) Draw indifference curves of either type of customer in \((r, m)\) space, and describe their properties (slope, curvature etc.).

(c) Draw the zero expected-profit locus for either type of customer in \((r, m)\) space.

(d) Draw the line of contracts that insure either type of customer perfectly in \((r, m)\) space.

(e) Derive first-best contracts for either type of customer, i.e., which maximize their expected utility subject to a zero profit constraint. Depict these diagrammatically in the \((r, m)\) space.

(f) Is the first-best allocation incentive compatible, if each customer has private information about which risk category she belongs to?
(g) Define a separating Spencian competitive equilibrium for this economy. Describe two such equilibria that happen to be Pareto-ordered.

(h) Define a pooling Spencian competitive equilibrium for this economy. Derive the entire set of such equilibria. Is there a pooling equilibrium which offers full insurance to both types of customers?

2. Consider the following signaling model. A consumer has a continuous utility function \( u(\cdot) \) over wealth, where \( u' > 0 \) and \( u'' < 0 \). The consumer has initial wealth \( w > 0 \). A low risk consumer has an accident probability \( \pi_L \), and a high risk consumer has an accident probability \( \pi_H \) where \( \pi_L < \pi_H \). The loss in an accident is \( L \). The consumer and insurance company play an extensive form game. First, nature chooses the consumer’s type. With probability \( \alpha \) the consumer is high risk, and with probability \( (1-\alpha) \) the consumer is low risk. The consumer learns his type, but the insurance company does not. Second, the consumer proposes a premium \( p \). Third, the insurance company decides whether to accept or reject the proposal. If the insurance company accepts, the consumer pays \( p \) to the insurance company, and the insurance company pays \( L \) to the consumer in the event of an accident. Finally, nature decides whether the consumer has an accident, and payoffs are realized. A pure strategy perfect Bayesian equilibrium of this game is described by \((p_L, p_H, \beta(p), \sigma(p))\). Here \( p_L \) and \( p_H \) are the premiums proposed by the low- and high-risk consumer, \( \beta(p) \) is the insurance company’s belief that the consumer is low risk, and \( \sigma(p) \in \{A, R\} \) is the insurance company’s acceptance/rejection decision.

(a) Define the concepts of separating and pooling equilibria for this game.

(b) Show that there are no separating equilibria in which both types of consumers purchase insurance.

(c) Characterize the pooling equilibria.