Ec 703 Microeconomic Theory
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Answer all three questions. Answers should be complete, explaining all steps in the reasoning, while omitting irrelevant details. There is no need to repeat the statement of the question in your answer.

1. [3*6 = 18 marks] Consider an exchange economy where all agents share a common utility function, which takes either of the following forms:

   (a) \( U = \prod_{i=1}^{L} x_i^{\alpha_i} \)

   (b) \( U = \sum_{i=1}^{L} a_i [x_i]^p \)

   (c) \( U = \min_{i=1}^{L} \left\{ \frac{x_i}{\alpha_i} \right\} \)

In each case provide (the weakest possible) restrictions on parameters (of the utility function, endowments) that guarantee (i) existence, and (ii) uniqueness of a Walrasian equilibrium. [Provide complete arguments. You are free to invoke theorems discussed in class. In case (c) where there may be multiple utility maximizing bundles involving differing quantities of some good, assume that agents demand the bundle with the smallest quantity.]

2. [6*2=12 marks] Are the following statements true or false (for an exchange economy where every agent has continuous, monotone and convex preferences)? If true under additional assumptions, specify those assumptions; if false outline an example where the statement is violated. There is no need to provide detailed proofs, simply outline the argument in two or three sentences at most for each part.

   (i) Every core allocation is Pareto optimal.
(ii) Every Pareto optimal allocation is a core allocation.

(iii) Every core allocation is Walrasian.

(iv) Every core allocation satisfies the equal treatment property.

(v) Every core allocation in a \( N \) replica economy is also a core allocation in a \( N + 1 \) replica economy.

(vi) Every core allocation in a \( N + 1 \) replica economy is also a core allocation in a \( N \) replica economy.

3. [2+3+2+3+8+2=20 marks] Consider an exchange economy with one physical consumption good, and \( I \) households, where each household faces a risk of an accident. In the absence of an accident a household’s endowment is \( \omega \); with an accident the endowment is 0. Each household faces an identical and independent accident probability \( p \in (0,1) \), known to each of them. They all have logarithmic utility.

(a) How many states of nature are there in this economy?

(b) Derive the set of \textit{ex ante} Pareto optimal allocations.

(c) Derive the set of \textit{ex post} Pareto optimal allocations.

(d) Suppose there is a complete set of Arrow securities traded before the state of nature is realized. Derive the demand function for these Arrow securities for any given household.

(e) Show that the equilibrium price of an Arrow security for state \( s \) is proportional to \( \frac{\pi_s}{W_s} \), where \( \pi_s \) denotes the probability of state \( s \) and \( W_s \) the aggregate endowment in that state. Interpret this result in economic terms.

(e) Verify that the equilibrium allocation of this economy is \textit{ex ante} Pareto optimal, using your answer to (b) above.