1. Consider an exchange economy with one consumption good, $S$ states of nature, and $I$ households, where household $i$ has endowment $\omega_{is} > 0$ in state $s$, beliefs $\pi_{is}$ over states of nature, and a constant relative risk aversion of $\rho > 0$ ($\rho$ is the same for all households). Consumption in each state is constrained to be nonnegative. Before the state of nature is known, there is a competitive market for Arrow securities corresponding to each state $s$.

(a) Derive the excess demand for security corresponding to any given state $s$ of any given household (given prices $q_s$ for the state $s$ security, $s = 1, \ldots, S$).

(b) Check whether these demand functions satisfy the five conditions that guarantee existence of a competitive equilibrium on the securities market.

(c) Check whether these demand functions satisfy some set of sufficient conditions for uniqueness of the competitive equilibrium.

**Bonus points:** (d) Assuming that all households hold identical beliefs, show that the equilibrium price of the state $s$ security is proportional to $\frac{\pi_s}{\sum_i \omega_{is}}$.

2. For any given number $n$ less than $I$ the number of consumers, define the $n$-core of an exchange economy to be the set of allocations that are not blocked by any coalition containing $n$ or fewer number of consumers. Prove that the Walrasian allocation is in the $n$-core, for any value of $n$. 


3. Consider an exchange economy with one physical good, $S$ states of nature, and $I$ households, where all households share the same beliefs, and the same state independent von-Neumann Morgenstern utility function which has constant absolute risk aversion. The aggregate endowment of the economy (i.e., the endowment of the single good aggregated across all households) depends on the state of nature. Show that in any *ex ante* Pareto optimal allocation, the consumption of any individual $i$ in state $s$ deviates from per capita consumption $\bar{c}_s$ in the economy by a constant, i.e., $c_{is} - \bar{c}_s = k, s = 1, \ldots, S$ for some $k$ which is independent of $s$. 