1. Consider an economy with $I$ households and $J$ firms. Each firm has a strictly convex and compact production set. Each household has (i) a given ownership share $\theta_{ij}$ in firm $j$, (ii) strictly monotone preferences, (iii) an excess demand function $Z^i(p)$ defined for all $p > 0$, incorporating incomes resulting from $i$'s share in different firms' profits. $Z^i(.)$ is continuous, homogeneous of degree zero, bounded from below and unbounded above (i.e., with respect to the maximum excess demand across all commodities, as some price tends to zero). Show that the economy has at least one competitive equilibrium.

2. Consider an exchange economy with $L$ commodities where for every household $i$ there is a good $l(i)$ on which household $i$ spends all its income. Under what additional conditions will competitive equilibrium in this economy be unique?

3. An exchange economy has two dates $t = 0, 1$ and two states of nature $s = 1, 2$ which will be revealed at date 1. Use $s = 0$ to denote the date-event pair corresponding to date 0. There is one physical commodity, and two consumers $i = 1, 2$ whose endowments $\omega_{is}$ are as follows: $\omega_{10} = 2, \omega_{11} = 4, \omega_{12} = 3, \omega_{20} = 4, \omega_{21} = 2, \omega_{22} = 3$. Both share the von-Neumann-Morgenstern utility $\log c_0 + \log c_1$, where $c_t$ denotes date $t$ consumption. Consumer 1 believes $s = 1$ with probability $\frac{3}{4}$, while consumer 2 believes $s = 1$ with probability $\frac{1}{4}$. At date 0, consumers trade in the commodity, besides two assets $k = 1, 2$ whose date-1 returns $r_{sk}$ are given by $r_{11} = 1, r_{12} = 2, r_{21} = 0, r_{22} = 1$. At date 1, spot commodity markets open.

(a) Derive the entire set of ex ante Pareto optimal allocations in this economy. Are these allocations ex post Pareto optimal as well?

(b) Describe carefully the optimization problem defining the optimal asset demands of the two consumers at date 0 (You need not derive the asset demand functions: show the objective function and the budget constraints.)

(c) What can you say about existence and Pareto optimality of Radner equilibria in this economy?