1. An economy has two assets, one safe and one risky. The safe asset generates income \( r \) per dollar invested, while the risky asset generates income \( R_i \) per dollar invested in state \( i \). There are three states: \( i = L, M, H \) that are equally likely ex ante, where \( R_H > R_M = r > R_L > 0 \).

There are three kinds of investors: \( j = L, H, U \). Proportion \( \lambda \) of them are \( L \) type investors, who receive a signal which tells them whether the state is \( L \) or not \( L \). Proportion \( \lambda \) are \( H \) type investors, who receive a signal which tells them whether the state is \( H \) or not \( H \). The remaining investors are of the U-type, who receive no signal. All investors are otherwise identical: each has an initial endowment of one safe and one risky asset. All investors are price-takers and decide on their trades after seeing the market price and the realization of their private signals.

(a) Represent the information available to each type of investor in the form of information partitions of the set of states, and also by a signal function mapping states into signals.

(b) Define a rational expectations equilibrium (REE) for this particular economy. (Please do not provide a more general definition).

(c) Derive a REE for this economy (compute the equilibrium prices and trades).

2. There are two types of blood donors: altruists (A-type) and desperados (D-type) in equal proportions. Each A-type donates one unit of blood, regardless of the price. Each D-type has a utility function \( m_s - \frac{b^2}{2} \), where \( m_s \) is units of money earned from blood sales, and \( b \) is the quantity of blood sold. The blood of the D-types is of inferior quality to that donated.
by the A-types. The type of a blood supplier is known privately to that supplier. Let \( \lambda \) denote the fraction of blood supplied on average by the A-types. Buyers of blood have the following utility function over quality \( \lambda \) and quantity of blood \( b \) purchased, and the money \( m_b \) spent: \( \lambda \log b - m_b \). There are as many buyers of blood as there are suppliers of blood. All are price-takers.

Define and describe a competitive equilibrium of this adverse selection economy. Is the equilibrium price unique?

3. There are two types of workers in an economy with productivity \( \theta_L \) and \( \theta_H \) respectively, where \( \theta_H > \theta_L > 0 \), and \( \lambda \in (0, 1) \) is the fraction of workers with productivity \( \theta_H \). All workers have zero reservation wages. Each worker also selects a level of training \( t \), and a worker with productivity \( \theta \) has a utility \( w - \frac{t}{\theta} \), where \( w \) is the wage received. There are a large number of firms that engage in Bertrand competition with one another on the labor market; all of them have constant returns to scale and maximize expected profit. The profit of any firm from hiring a worker with productivity \( \theta \) at wage \( w \) is \( \theta - w \).

(a) Represent workers preferences by indifference curves in \((w, t)\) space. Do they satisfy the single crossing property?

(b) Consider the following three allocations:

(A) \( t_L = \frac{(\theta_L)^2}{2}, w_L = \theta_L, t_H = \theta_H(\theta_H - \theta_L) + \frac{(\theta_L)^2}{2}, w_H = \theta_H. \)

(B) \( t_L = t_H = 0, w_L = w_H = \lambda \theta_H + (1 - \lambda) \theta_L. \)

(C) \( t_L = 0, w_L = \theta_L, t_H = \theta_H(\theta_H - \theta_L), w_H = \theta_H. \)

Which of these three allocatons are Perfect Bayesian Equilibrium outcomes of the signaling game (where workers move first by selecting levels of training, then firms make wage offers)? Explain your answer fully: if they are not PBE outcomes explain why; otherwise construct the associated equilibrium strategies and beliefs.

(c) Which of the above three allocations are subgame perfect Nash equilibrium outcomes of the screening model (where firms move first by offering a wage policy \( w(t) \), and then
workers select a firm and a training level)?

4. A monopoly venture capitalist $V$ provides finance to an entrepreneur $E$ to start a project. If $V$’s investment in the project is $s$ dollars, the returns from it are $\theta s - \frac{s^2}{2}$, while $V$’s financing cost is $cs$ dollars. $E$ knows the parameter $\theta$, which takes two possible values $\theta_L, \theta_H$, where $\theta_H > \theta_L > c$. $V$ has a prior that $\theta = \theta_H$ with probability $q$. Payoff functions are $\theta s - \frac{s^2}{2} - r$ for $E$, and $r - cs$ for $V$, where $r$ is the payment made by $E$ to $V$ after project returns are realized. Both have outside options equal to zero dollars. $V$ makes a take-it-or-leave-it contract offer to $E$.

   (a) What are the inequality constraints that characterize the set of feasible contracts?

   (b) Prove that the scale of the project must be nondecreasing in $\theta$.

   (c) Prove that in selecting an optimal contract $V$ can ignore the participation constraint for the $\theta_H$ type of entrepreneur.

   (d) Prove that the participation constraint of the $\theta_L$ type must bind in the optimal contract.