EC320 Solutions to Problem Set 4

(a) *Consider a household owning less than or equal to two acres of land, who decide to cultivate it themselves. How should they divide their time between working on the farm and working on the labor market? What is the maximum they can earn in total this way?*

- Households with 1 acre of land
  - 50 worker-hours per week per acre: produce $12 > $5
    100 worker-hours per week per acre: produce $5.5 > $5
    Therefore, they will put 100 worker-hours per week on the farm.
  - ∴ Total income=$1,375
    2 family members work full-time on 1 acre of land and earn $(6 + 2.75) * 50 * 2 = \$875$
    2 family members work either in local industry or another farm and earn $5 * 50 * 2 = \$500$
- Households with 2 acres of land
  - They will also put 100 worker-hours per week per acre.
  - ∴ Total income=$1,750
    4 family members work full-time on 2 acres of land and earn $(6 + 2.75) * 50 * 4 = \$1,750$

(b) *Consider a household leasing less than or equal to two acres of land on a 50:50 sharecropping contract. How should they divide their time between working on the farm and working on the labor market? What is the maximum they can earn in total this way? How much will the landlord earn from the rental?*

The tenants will not put beyond 50 worker-hours per week per acre because if they do, they get $(2.75*2)\frac{1}{2} = \$2.75$ for an additional worker-hour, which is less than their outside option, $5.

- Households with 1 acre of land earn $1,300 each.
  - 1 acre sharecropped: $6 * 50 * 2 * \frac{1}{2} = \$300$
  - 4 family members working elsewhere $50 * 5 * 4 = \$1,000$
- Households with 2 acres of land earn $1,600 each.
  - 2 acres sharecropped: $6 * 50 * 2 * \frac{1}{2} * 2 = \$600$
  - 4 family members working elsewhere $50 * 5 * 4 = \$1,000$

(c) *Use your answers to (a) and (b) to show that any family owning less than or equal to two acres will prefer to cultivate it rather than lease it to someone else.*

For families with 1 or 2 acres of land, cultivating the land by themselves brings more total income than leasing it out to tenants. They will prefer to cultivate it by themselves.

(d) *Now consider a family owning x acres of land, where $x > 2$, who decide to cultivate it using hired labor and a supervisor, as explained above. How much will this family earn? For what values of x will the family earn positive profit?*

- Households will prefer hiring 100 worker-hours per week per acre to hiring 50 worker-hours per week per acre if $x \geq 20$
    - Option 1: hiring 50 worker-hours per week per acre
      Income$=((12-5)*x-10)*50 = 350x - 500$
    - Option 2: hiring 100 worker-hours per week per acre
      Income$=((12-5)x-10)*50 + ((5.5-5)x-10)*50 = 375x - 1000$
    - $350x - 500 \leq 375x - 1000$ if and only if $x \geq 20$
- Hiring labor yields a positive profit when $x \geq \frac{10}{7}$
    - $350x - 500 \geq 0$

(e) *Consider the same family as in (d), who now consider the option of cultivating 2 acres themselves, and leasing out the remaining x-2 acres. For what values of x would this be the better option than in (d)?*

- By cultivating 2 acres by themselves leasing out the remaining land, they earn \$$(300x + 1150)$
    - As in the case where the household owns 2 acres of land in (a), by self-cultivating 2 acres, the family earns \$1,750 from the 2 acres.
    - Based on the reasonings in (b), by leasing out *x-2* acres of land to tenants, the family earns $300*(x-2) = \$(300x - 600)$
- Now to compare (d) and (e), consider two cases where (1) $x \leq 20$ where households hire 50 worker-hours per week per acre in (d) and (2) $x \geq 20$ where they hire 100 worker-hours per week per acre in (d).
    (1) $x \leq 20$
      For (e) to be more desirable than (d), the following should hold:
      $$300x + 1150 \geq 350x - 500$$
      $$\therefore x \leq 31$$
    (2) $x \geq 20$
      For (e) to be more desirable than (d), the following should hold:
      $$300x + 1150 \geq 375x - 1000$$
      $$\therefore x \leq 28\tfrac{2}{3}$$
- Therefore, for $x \leq 28\frac{2}{3}$, (e) is better, and for $x \geq 28\frac{2}{3}$, (d) is better. In particular, in the latter, the family will want to hire 100 worker-hours per week per acre.

(f) *Finally, calculate productivity (income per acre) of land owned by a household with x units of land, across all possible values of x, when for each value of x the family chooses the best amongst the various options considered above. What does this imply about the effects of a land reform which redistributes land from medium and big landowners to landless families on average agricultural productivity in the region.*

x	Mode	Productivity per acre
1, 2	Self-cultivating	$875 - 500 = \$375$
3, ..., 28	Self-cultivating (2 acres) + 50:50 Sharecropping (x-2 acres)	$\frac{2*375+(600-250)*(x-2)}{x} = 350 + \frac{50}{x}$
29, 30, ...	Hiring labor	$\frac{((6+2.75)*50*20-100*50)*x-10*100-5*50*4}{x} = 375 - \frac{2000}{x}$

The above table summarizes the results where productivity is the money value of output net of opportunity cost. It is shown that the selected mode of cultivation varies with size of land owned. Small farms are family farms, intermediate sized ones are leased out, and large ones are capitalistic. Moreover, productivity and profitability depends on which mode is selected.

The effect of land redistribution will therefore depend on the nature of the redistribution. A redistribution which gives each landless worker up to 2 acres will unambiguously increase agricultural productivity and

aggregate income in the region (irrespective of whether this comes from intermediate or large size farms). But if each landless worker is given more than 2 acres of land each, with this land coming from breaking up the large capitalist farms, this will cause a switch into a combination of family labor cultivation and tenancy. Since tenant farms are the least productive, this may lower productivity and aggregate income overall.