Solutions to Problem Set No. 2

(a) What is the wage and average product of labor in agriculture in Year 1? What is its marginal product? What proportion of the rural labor force constitutes surplus labor?

In Year 1, there are only ten factories, which can employ a maximum of 100 workers. Hence most farms will still have all five siblings working there. The total farm income will be $100, so the average product of labor is $20 = \frac{100}{5}$. The marginal product is 0, since when there are at least three siblings working on the farm, any additional worker does not increase farm income. Consequently two out of five siblings do not contribute anything incrementally to farm income; the proportion of rural labor force which is surplus is 40%.

(b) What are the wage, the average and the marginal products of labor in industry? Is there any surplus labor in industry? How many workers does each factory employ? What is the fraction of the aggregate labor force in the economy in industrial employment in Year 1? What is the aggregate profits made by entrepreneurs, and the national income in Year 1 (the latter defined by the sum of profits, and wages net of cost-of-living).

Under the assumptions of the Lewis model, industry would have to offer the same real income to each worker as they receive in agriculture. Since the average income of each agricultural worker is $20, and the cost of living differential of the city is $5, industry would have to offer a wage of at least $25 to attract workers. Moreover, at this wage, the number of workers willing to work in the city will be at least 20,000, since this is the amount of surplus labor in the countryside, far in excess of the maximum number of jobs available in factory employment, i.e., 80. Hence the factory wage will settle at $25, since entrepreneurs can get all the labor they need at this wage.

Given this wage, each factory will employ exactly eight workers, since this will maximize its profits. (The eighth worker contributes $30, while the ninth contributes $20 to factory
revenues, so the former contributes more than its wage, while the latter contributes less.) At this level of employment, the marginal product of one additional worker is positive. Hence there is no surplus labor in industry.

Factory employment in Year 1 would therefore be exactly 80. The aggregate labor force in the economy is 50,000, so the fraction of labor force employed in industry is 0.16%.

Each factory earns profit equal to its revenues $520, less wage costs $200 = 8 \times 25$, i.e., $320. The ten factories will cumulatively earn an aggregate profit of $3200.

Each of the 80 workers earns a net wage of $20, so aggregate net wages in industry amount to $1,600. Total farm income equals $10,000 \times $100 = $1 million. Hence national income in Year 1 is the sum of farm income, industrial wages and profits, amounting to $1,004,800.

(c) How many factories will there be in Year 2? Describe what happens to the rural wage, the urban wage, employment in industry, and national income in Year 2.

In Year 1, total profits were $3200, of which half would be reinvested in new factories. Since each factory costs $160 to set up, ten new factories will be set up by the beginning of Year 2. At the going wage of $25, each factory would still want to employ eight workers. So factory employment will double to 160 if the urban wage remains equal to $25, whence there is still an excess supply of labor from the countryside. So the industry wage in Year 2 will remain at $25, and the rural wage will also remain at $20 (given Lewis’s assumption that it remains institutionally determined as long as it remains below the marginal product of labor).

Total profits would double to $6400. Total wages of industrial workers would amount to $3200. Total farm income would remain unchanged at $1 million, owing to continuation of surplus labor. Hence national income in Year 2 will be $1,009,600.

(d) What will happen in Year 3?

In Year 3 there will be 40 factories, with total factory employment amounting to 320 =
8 × 40. Wage rates will remain unchanged: $25 in the urban area, and $20 in the rural area. Per factory profit will be $320, as before, so total factory profit will be $12,800. National income will be $1,019,200.

(e) Describe (verbally and briefly) the main qualitative features of the first phase of this growth process. Can you calculate how many years it will last? What will be the main qualitative changes in the economy once the first phase is over?

A similar story will be repeated as long as surplus labor persists in agriculture. Every year the number of factories will double, since half the profits of any factory which are invested, exactly equal the price of a new factory. Every year there continues to be surplus labor in agriculture (since initially the extent of surplus labor equalled 20,000), during this first phase, until there are enough factories to absorb it all. During this phase, wages will remain unchanged, profits will increase, and so national income will increase.

The first phase will end after the eighth year. In Year 9, there will be 2560 factories, and the number of factory jobs available will equal 20,480, causing surplus labor in agriculture to disappear for the first time. Following this the economy will transit to the third phase of the Lewis process. The second phase does not arise because food is imported at the fixed world price throughout, so no food shortages ever arise.

Once the surplus labor disappears, the marginal product of labor will rise, and so will wages. Profit rates will fall, and with it, growth rates. So the economy slows down overall, but income is more equally distributed.
DETAILED CALCULATION OF THE OUTCOMES FOR DIFFERENT YEARS

Below I present detailed calculations of the outcomes for the first eleven years, in case you are interested (I do not assume that you can or have done this, so this is purely for those who curious to see how the whole things works, down to the last teeny detail). The table below presents the relevant numbers.

EVOLUTION OF THE ECONOMY, YEARS 1 TO 11

<table>
<thead>
<tr>
<th>Year</th>
<th>Rural Wage</th>
<th>Number Of Factories</th>
<th>Urban Wage</th>
<th>Per Factory Empl.</th>
<th>Per Factory Profit</th>
<th>Total Factory Empl.</th>
<th>Total Factory Profit</th>
<th>National Income</th>
<th>Growth Rate</th>
<th>Percent Labor in Industry</th>
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Upto Year 8 the first phase continues in the same manner: total factory employment is still less than 20,000, the amount of workers that would need to be taken out of agriculture for the labor surplus there to disappear.

In Year 9, the following happens. Factories demand more than 20,000 workers for the first time, and so all the surplus labor in agriculture disappears. Moreover, there is additional demand for 480 workers in the factory sector. At this stage all family farms have three siblings left still, and the marginal product of the third sibling equals $20, exactly equal to the urban wage less cost-of-living. So third siblings are also willing to work in a factory at the going urban wage of $25. They are equally willing to stay on in the farm, with the rural wage still at $20. The extra 480 jobs will therefore be filled by third siblings from 480 farms, while those on remaining farms will stay behind.
Hence in Year 9, also, urban wages will stay at their previous level, and so will per-factory profit. (Think what would happen if the urban wage were to rise a bit above $25, say to $26. Then the third siblings on all farms would apply to the factories: there would be 10,000 applicants for 480 remaining jobs. Firms would react to this excess supply by lowering wages, and they would come down again to $25).

In Year 10 there will be twice as many factories as in Year 9. If the urban wage were to stay at its previous level, i.e., $25, then factories would seek to employ 40,960 workers. But if this many people were to leave agriculture, some farms would not be left with any siblings. Then the marginal product of labor on those farms would be $50, far in excess of the existing wage. According to the Lewis assumption, agriculture will get commercialized owing to the scarcity of labor in the countryside, and the rural wage would rise to $50. Then factories would have to offer $55 to induce people to work for them. At such a high wage, factories would find it profitable to employ fewer than eight workers each: indeed the profit maximizing employment level would be 5. Then total demand for labor in the factory sector would shrink to $31,000 = 5 \times 5120$, which is less than the number of applicants (50,000). So the wage cannot rise to as high as this.

Where will it settle? You can work through the implications of all possible wages between $25 and $55, calculate supply and demand for labor in the cities, and figure out where the market clears. The answer is: when the urban wage is $35, and the corresponding rural wage is $30, exactly four siblings per family farm will be looking for factory jobs (since this is the marginal product of the second sibling on the family farm). So there will be 40,000 applicants. Factories will seek to employ 7 workers each at this wage, yielding a total demand for 35,840 = 5120 \times 7 workers. So many applicants will be accepted, then, and the remainder will be content to go back home to their farm and earn the same real wage of $30.

In Year 10, therefore, the urban wage will be $35, total factory employment will be 35,840, each factory will earn a smaller profit of 490 – (7 \times 35) = $245. Total industry profit will be $245 \times 5120 = $1,254,400. Total industry wages will be (net of cost of living): $30 \times 35840 = $1,075,200. Total farm income is $624,800 = (10,000 \times 50) + (4,160 \times 30). National income thus equals $1,254,400 + 1,075,200 + 624,800 = 2,954,000. This year the economy has entered the phase of labor scarcity, rising wages, and falling profit per factory: this is the neoclassical phase when the economy has a shortage rather than surplus of labor, and marginal product of labor (net of cost of living) is equalized across agricultural and industrial sectors (at $30).

Year 11 will see 3920 = $4,254,000 \div 2.100 new factories, yielding a total of 9040 factories. The urban wage must rise beyond the level of previous year, since otherwise the demand for workers 7 \times 9040 would
exceed the total number of workers (50,000) in the entire economy. Check that the equilibrium urban wage this year will be $55, with each factory employing five workers each, total factory employment being 45,200 = 9040 x 5. The corresponding rural wage and marginal product will be $50. So 5,200 farms will not have any people left; the remaining ones will have exactly one sibling left. Total farm income will decline to $260,000 = 5200 x 50!

Each factory will earn a profit of $125 = 400 - {5 x 55}. So total industry profit will be $1,130,000 = 9040 x 125. Industry wages will aggregate to 45,200 x $50 = $2,260,000, net of cost-of-living. National income in Year 11 will thus be $3,650,000 = 260,000 + 2,260,000 + 1,130,000.

There will be new factories built again in Year 12: indeed, under our preceding assumptions, entrepreneurs will seek to set up over 3000 new factories (investible funds of $565,000, at $160 per factory). The demand for factory employment will rise to above 50,000, the aggregate labor force in the entire economy, at the previous wage of $55. So the wage must increase further, whence the farm sector will be entirely abandoned! From this point onwards, all workers will be in industry, and the economy will specialize completely in manufacturing (and become a food importer). Wages will be bid up progressively, profits will decline. Growth will stop once profits are driven down to zero (and the wage rate is bid up to $100, with every factory eventually hiring one worker each).

During this second, scarce labor phase, growth is described by the Solow neoclassical model quite well: it is driven by saving and investment, with progressive capital deepening (fewer workers per factory) and rising wages. The transfer of workers from unproductive agricultural occupations to productive industrial jobs is no longer a source of growth, unlike the first phase — this is because labour is equally productive in both occupations. Given the absence of technological progress or human capital investments, growth must ultimately cease.

Note in the previous table how growth rates initially accelerate, then slow down. Upto Year 9, almost all the benefits of growth accrue to the industrial capitalists: only after this do factory workers gain. Eventually, profits will be driven down to zero, and workers will receive all of national income. In Marxian terms, justice shall finally prevail!