

SOLUTIONS TO PROBLEM SET 2

The economy of Lorenzini has ten million people and two sectors (traditional and modern). In the year 1990, three million people live in the modern sector and earn \$2000 per month. The rest live in the traditional sector and earn \$1000 per month. As a result of new investments in the modern sector during the 1990s, two million people moved out of the traditional sector by year 2000 to take up new jobs in the modern sector. Per capita incomes within each sector did not change throughout the decade.

(a) Graph the Lorenz curves for 1990 and 2000 respectively. (Plot cumulative income shares of successive deciles, i.e., the poorest 10%, 20%, 30% etc. of the population, and connect successive points with a straight line)

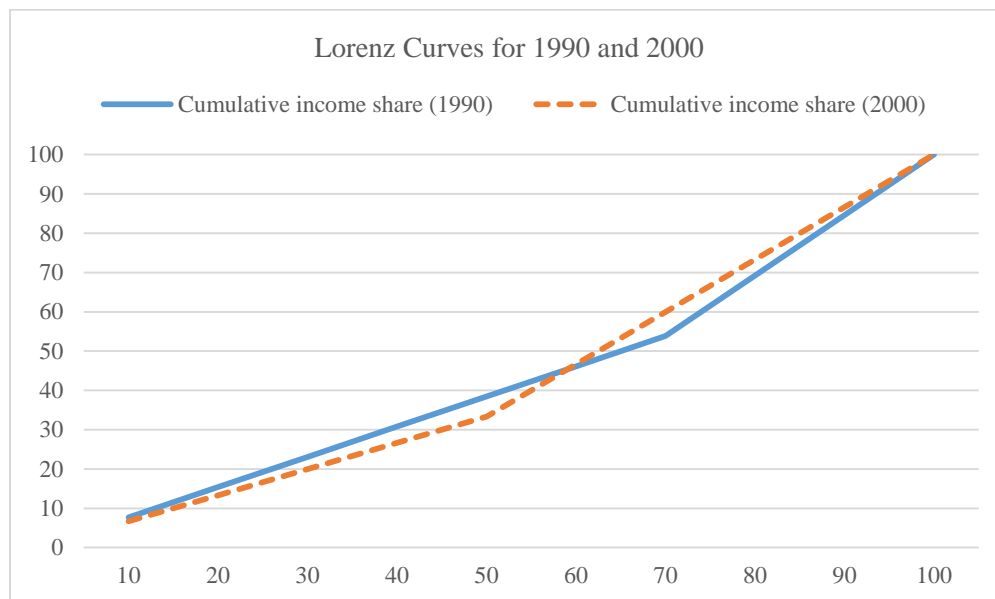
Lorenz curves for the two periods are obtained by plotting cumulative income shares against (poorest) population shares. They cross because the income share of the bottom 50% of the population has decreased while that of the bottom 60%, 70%, etc. of the population has increased.

1990

Poorest population share	10	20	30	40	50	60	70	80	90	100
Cumulative income	10000	20000	30000	40000	50000	60000	70000	90000	110000	130000
Cumulative income share (%)	7.7	15.4	23.1	30.8	38.5	46.2	53.8	69.2	84.6	100.0

2000

Poorest population share	10	20	30	40	50	60	70	80	90	100
Cumulative income	10000	20000	30000	40000	50000	70000	90000	110000	130000	150000
Cumulative income share (%)	6.7	13.3	20.0	26.7	33.3	46.7	60.0	73.3	86.7	100.0



(b) Calculate the Gini coefficient (G) and coefficient of variation (CV) of the income distributions in 1990 and 2000 respectively.

- Gini coefficient (G)

There are several ways to calculate the Gini coefficient. One of the options is to look at the pairwise income disparities.

$$\begin{aligned} G &= \frac{\text{Mean pairwise differences in consumption}}{\text{twice average consumption}} \\ &= \frac{\text{Sum of ALL pairwise consumption disparities}}{(\text{Size of total population})^2} \\ &= \frac{2 * \text{Total consumption}}{\text{Size of total population}} \\ &= \frac{\text{Sum of ALL pairwise consumption disparities}}{2 * \text{Total consumption} * \text{Size of total population}} \end{aligned}$$

Therefore, under the assumption that income equals consumption, we have

$$\begin{aligned} G_{1990} &= \frac{42000}{2 * 130000} = 0.1615 \\ G_{2000} &= \frac{50000}{2 * 150000} = 0.1667 \end{aligned}$$

- Coefficient of variation (CV)

Average income is 1300 in 1990 and 1500 in 2000. Therefore,

$$\begin{aligned} CV_{1990} &= \frac{((1000 - 1300)^2 * 0.7 + (2000 - 1300)^2 * 0.3)^{1/2}}{1300} = 0.353 \\ CV_{2000} &= \frac{((1000 - 1500)^2 * 0.5 + (2000 - 1500)^2 * 0.5)^{1/2}}{1500} = 0.333 \end{aligned}$$

(c) Discuss what the different inequality measures (the Lorenz curve, G and CV) indicate regarding the change of inequality between 1990 and 2000. If they do not provide similar answers, explain why.

It is not clear which of the two Lorenz curves is farther away from the 45 degree line (perfect equality). More importantly, the curves cross, and it is known that G and CV might give different evaluations in this case. From (b), $G_{1990} < G_{2000}$, which means income inequality over the whole population has become worse. On the other hand, $CV_{1990} > CV_{2000}$, which we can interpret that the income over the nation was more dispersed. The two inequality measures disagree.

(d) Suppose the poverty line is \$1500 per month. Compute the head-count ratio and poverty gap ratio before and after the change.

$$\begin{aligned} PHCR_{1990} &= \frac{7}{10} = 0.7 \\ PHCR_{2000} &= \frac{5}{10} = 0.5 \\ PGAP_{1990} &= \frac{(1500 - 1000)}{1500} * \frac{7}{10} = 0.233 \\ PGAP_{2000} &= \frac{(1500 - 1000)}{1500} * \frac{5}{10} = 0.167 \end{aligned}$$

The fraction of the population living under the poverty line becomes smaller in 2000, and poverty is less intense in 2000.

(e) Write a brief (verbal) assessment of the development experienced by this economy between 1990 and 2000, based on changes in the following development indicators: per capita income, inequality and poverty.

- As more people work in the higher-income sector, per capita income increases from 1300 in 1990 to 1500 in 2000. Therefore, the country on average is richer in 2000.
- Whether the movement of workers alleviates or aggravates inequality is inconclusive. One might say that since G has increased inequality has been worsened within the 10 years. However, CV has decreased, which leads to an opposite conclusion that there was an improvement in inequality.
- Poverty head-count ratio has decreased, which means there are less people (relative to the total population) who experience extreme poverty. In addition, based on poverty gap, poverty is less severe in 2000.
- In summary, according to the measures we have, 1) the economy of Lorenzini has grown in size (per capita income has increased with population unchanged) and 2) the fraction of population living under the poverty line has decreased, and poverty has become less intense. However, 3) whether or not there was an improvement in inequality cannot be determined.