

Ec 320 2014: Solutions to Homework Problem Set 1

September 23, 2014

1 Solutions to HW 1

1. Current capital productivity α is 0.5. As long as p.c.i. is below \$2000, the capital productivity will remain at this value. So the Harrod-Domar growth formula can be applied with $\alpha = 0.5$ until p.c.i. gets to \$2000. After that the capital productivity will fall to $(0.5) * (0.9) = 0.45$, until p.c.i. gets to \$3000. Hence over the range of p.c.i. from \$2000 to \$3000, the Harrod-Domar growth formula can be applied with $\alpha = 0.45$. And so on.

From 1500 ~ 2000: $\alpha = 0.5$

$$\Rightarrow g = 8 * (0.5) - 3 = 1\%$$

From 2000 ~ 3000: $\alpha = 0.5 * 0.9$

$$\Rightarrow g = 8 * (0.5) * (0.9) - 3 = 0.6\%$$

From 3000 ~ 4000: $\alpha = 0.5 * (0.9)^2$

$$\Rightarrow g = 8 * 0.5 * (0.9)^2 - 3 = 0.24\%$$

From 4000 ~ 5000: $\alpha = 0.5 * (0.9)^3$

$$\Rightarrow g = 8 * 0.5 * (0.9)^3 - 3 = -0.08\%$$

Hence g the growth rate of p.c.i. is slowing down as it rises above \$2000, owing to the fall in the productivity of capital (as in the Solow model). When it is in the range from \$3000-4000, it will be growing very slowly at .24%. Once it goes above \$4000, the growth rate becomes negative. This means p.c.i. will fall to \$4000, and stay at that level thereafter. In the long run the growth rate will be 0, and per capita income will stabilize at \$4000.

Figure 1 shows the evolution of per capita income over time.

Problem 1



Notes: Per capita income is 1500 at the initial period (period 0) and grows at the rate of 1%, reaching 2001.756 at period 29. After then the growth slows down; it starts growing at the rate of 0.6%. At period 97, it is 3006.587, and after then the growth rate decreases once again to 0.24%. At period 217, it reaches a level above 4000, 4008.674. Since the growth rate is -0.084% for any level above 4000 (below 5000), the PCI level immediately decreases. At period 220, it falls below 4000, which means it should rise again at the rate of 0.24%. In consequence, the PCI level is stabilized around 4000 in the long run.

Figure 1: Time Path of Per Capita Income in Problem 1

2.

- (i) The investment (equal to saving) rate will fall. Hence the new steady state level of p.c.i. corresponding to the lower saving rate will be below the current p.c.i. (which is the steady state corresponding to the previous saving rate). In other words, the current capital per worker is higher than the new steady state (p.c.i.) capital per worker. The economy will then experience negative growth of p.c.i. and capital per worker: both will shrink and converge to the new steady state. In the long run the p.c.i. level will be lower, while the long run growth rate will be zero (hence will remain unchanged).
- (ii) Since the current p.c.i. is below the steady state level, p.c.i. will be growing in the short run (before the arrival of the immigrants). The one-shot increase in population will result in a one-shot decrease in capital per worker, hence also in p.c.i. level. Thereafter the economy will start growing again, from a lower capital per worker. The short-run growth rate will be higher than what it is currently. The long-run p.c.i. level and growth rate will be unaffected.
- (iii) The economy is currently growing at the rate of growth of TFP, since it is currently in steady state. If the rate of TFP growth rises, the new steady state growth rate rises, and the economy will then be growing at this rate. Hence p.c.i. levels and growth rates rise both in the short and long run.

Figure 2 shows how the time-path of p.c.i. is altered in each of the above three cases.

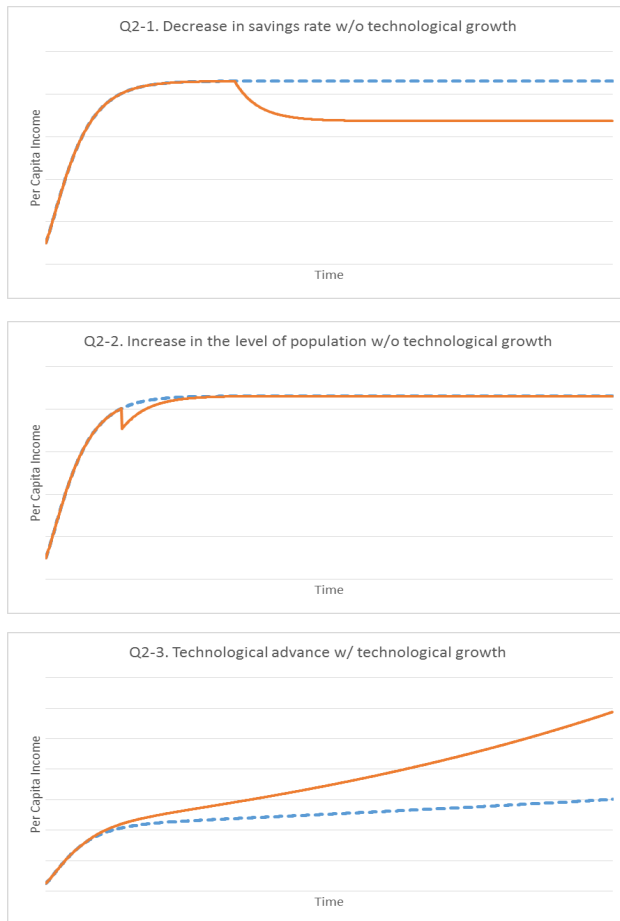


Figure 2: Time Path of Per Capita Income Before and After Each Change in Problem 2