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## Homework Problem Set No. 4

A certain agricultural region contains many families with varying amounts of land. A large number of them are landless, while others have land varying in size from 1 acre to 50 acres. The region also has a local industry, which offers work at a fixed wage rate of $\$ 5$ an hour. Its demand for labor is high enough that it is willing to hire any number of part-time workers, on a per hour basis.

The land can be used to grow a staple crop, whose market price is $\$ 2$ a bushel. The technology is such that one acre of land yields an average weekly output of 6 bushels per worker-hour, upto 50 worker-hours per week. Beyond this an additional 50 worker-hours per week can be applied, yielding incremental output at the (lower) rate of 2.75 bushels per worker-hour. Any further application of labor (beyond 100 hours per week) will not generate any additional output.

Each family has four able bodied people available to work, and each of them can work upto 50 hours per week, either in farming, or in the local industry. Every family owning land has three possible options: (i) cultivate it themselves with their own labor, (ii) lease it out to a tenant under a standard 50:50 sharecropping contract, or (iii) cultivate it on the basis of hired labor. Hired workers, however, need to be supervised. A single supervisor can be hired at $\$ 10$ per hour, and (unlike the assumption made in class) can supervise any number of workers at the same time. All the family members will then be required to oversee and manage the hired labor farm, and will not have time available to work on the labor market.
(a) Consider a household owning less than or equal to two acres of land, who decide to cultivate it themselves. How should they divide their time between working on the farm and working on the labor market? What is the maximum they can earn in total this way?
(b) Consider a household leasing less than or equal to two acres of land on a $50: 50$ share-
cropping contract. How should they divide their time between working on the farm and working on the labor market? What is the maximum they can earn in total this way? How much will the landlord earn from the rental?
(c) Use your answers to (a) and (b) to show that any family owning less than or equal to two acres will prefer to cultivate it rather than lease it to someone else.
(d) Now consider a family owning $x$ acres of land, where $x>2$, who decide to cultivate it using hired labor and a supervisor, as explained above. How much will this family earn? For what values of $x$ will the family earn positive profit?
(e) Consider the same family as in (d), who now consider the option of cultivating 2 acres themselves, and leasing out the remaining $x-2$ acres. For what values of $x$ would this be the better option than in (d)?
(f) Finally, calculate productivity (income per acre) of land owned by a household with $x$ units of land, across all possible values of $x$, when for each value of $x$ the family chooses the best amongst the various options considered above. What does this imply about the effects of a land reform which redistributes land from medium and big landowners to landless families on average agricultural productivity in the region.

