

Solutions to Problem Set No. 3

(a) *Graph the Lorenz curves before and after the change and check that they cross. (Plot cumulative income shares of successive deciles, i.e., the poorest 10%, 20%, 30% etc. of the population, and connect successive points with a straight line).*

See Graph 1.

(b) *Calculate the Gini coefficient and coefficient of variation of the two distributions in 1990 and 2000 respectively.*

Per capita income in 1990 and 2000 are:

$$\mu_1 = \frac{1}{n} \sum_{j=1}^m n_j y_j = \frac{7,000,000 \times 1,000 + 3,000,000 \times 2,000}{10,000,000} = 1300(\text{dollars})$$
$$\mu_2 = \frac{1}{n} \sum_{j=1}^m n_j y_j = \frac{5,000,000 \times 1,000 + 5,000,000 \times 2,000}{10,000,000} = 1500(\text{dollars})$$

The Gini coefficient in 1990 and 2000 are:

$$G_1 = \frac{1}{2n^2\mu_1} \sum_{j=1}^m \sum_{k=1}^m n_j n_k |y_j - y_k|$$

$$= \frac{(7,000,000 \times 3,000,000)|1,000 - 2,000| + (3,000,000 \times 7,000,000)|2,000 - 1,000|}{2 \times 10,000,000^2 \times 1300}$$

(When $j = k$, $|y_j - y_k| = 0$)

$$= 0.16$$

$$G_2 = \frac{1}{2n^2\mu_2} \sum_{j=1}^m \sum_{k=1}^m n_j n_k |y_j - y_k|$$

$$= \frac{(5,000,000 \times 5,000,000)|1,000 - 2,000| + (5,000,000 \times 5,000,000)|2,000 - 1,000|}{2 \times 10,000,000^2 \times 1500}$$

(When $j = k$, $|y_j - y_k| = 0$)

$$= 0.17$$

The coefficient of variation in 1990 and 2000 are:

$$C_1 = \frac{1}{\mu_1} \sqrt{\sum_{j=1}^m \frac{n_j}{n} (y_j - \mu_1)^2} = \frac{\sqrt{0.7(1000 - 1300)^2 + 0.3(2000 - 1300)^2}}{1300}$$

$$= 0.35$$

$$C_2 = \frac{1}{\mu_2} \sqrt{\sum_{j=1}^m \frac{n_j}{n} (y_j - \mu_2)^2} = \frac{\sqrt{0.5(1000 - 1500)^2 + 0.5(2000 - 1500)^2}}{1500}$$

$$= 0.33$$

(c) Discuss what the different inequality measures (the Lorenz curve, G and CV) indicate regarding the change of inequality between 1990 and 2000. If they do not provide similar answers, explain why.

These two measures give different answers to the question whether inequality rises or falls. The two Lorenz curves cross each other, which is why they give different answers. As people move from the traditional sector, it reduces the inequality between the movers

and those already in the modern sector in 1990, but it increases the inequality between the movers and those left behind in the traditional sector.

(c) *Suppose the poverty line is \$1500 per month. Compute the head-count ratio, poverty gap ratio and income gap ratio before and after the change.*

Head count ratio before and after the change are:

$$HCR_1 = \frac{7,000,000}{10,000,000} = 0.7$$

$$HCR_2 = \frac{5,000,000}{10,000,000} = 0.5$$

Poverty gap ratio before and after are:

$$PGR_1 = \frac{\sum_{y_i < p} (p - y_i)}{n\mu_1} = \frac{7,000,000 \times (1500 - 1000)}{10,000,000 \times 1300} = 0.27$$

$$PGR_2 = \frac{\sum_{y_i < p} (p - y_i)}{n\mu_2} = \frac{5,000,000 \times (1500 - 1000)}{10,000,000 \times 1500} = 0.17$$

Income gap ratio before and after are:

$$IGR_1 = \frac{\sum_{y_i < p} (p - y_i)}{pHC_1} = \frac{7,000,000 \times (1500 - 1000)}{7,000,000 \times 1500} = 0.33$$

$$IGR_2 = \frac{\sum_{y_i < p} (p - y_i)}{pHC_2} = \frac{5,000,000 \times (1500 - 1000)}{5,000,000 \times 1500} = 0.33$$

(d) *Write a brief (verbal) assessment of the development experienced by this economy between 1990 and 2000, based on changes in the following indicators: per capita income, inequality and poverty.*

Between 1990 and 2000, the income of 2 million people increased by \$1000. As a result, per capita income increased. But it is unclear how inequality has changed. The growth process increased the gap between those who moved out of the rural sector and those who remained there. But on the other hand it reduced the gap between those who moved and those who were in the urban sector to start with. It is difficult to say whether inequality

as a whole has gone up or not. The Gini coefficient and the coefficient of variation give different answers.

Poverty decreased, as measured by the head count ratio: fewer people lived below the poverty line after the change. The poverty gap ratio became smaller, implying that a smaller fraction of the economy's resources were needed in 2000 to eliminate poverty via redistributive transfers. The income gap ratio remained constant, on the other hand: the extent of poverty among those below the poverty line did not change.

Lorenz Curves

