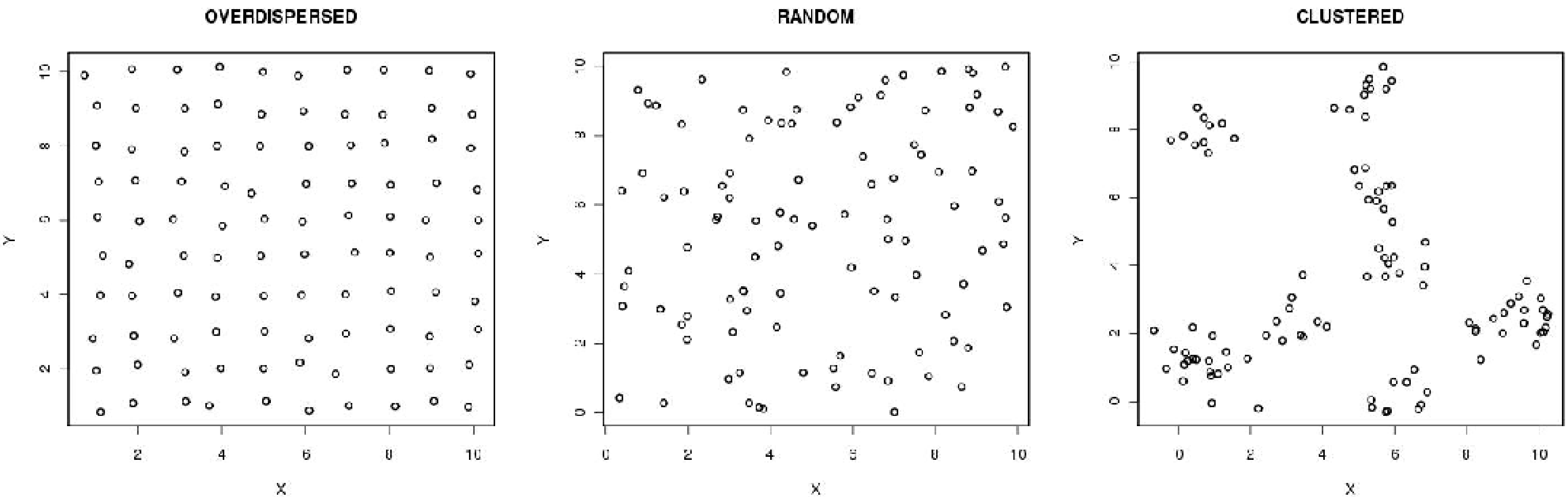
Spatial Data

Types of Spatial Data

- Point pattern
- Point referenced
  - “geostatistical”
- Block referenced
  - Raster / lattice / grid
  - Vector / polygon

Point Pattern Data

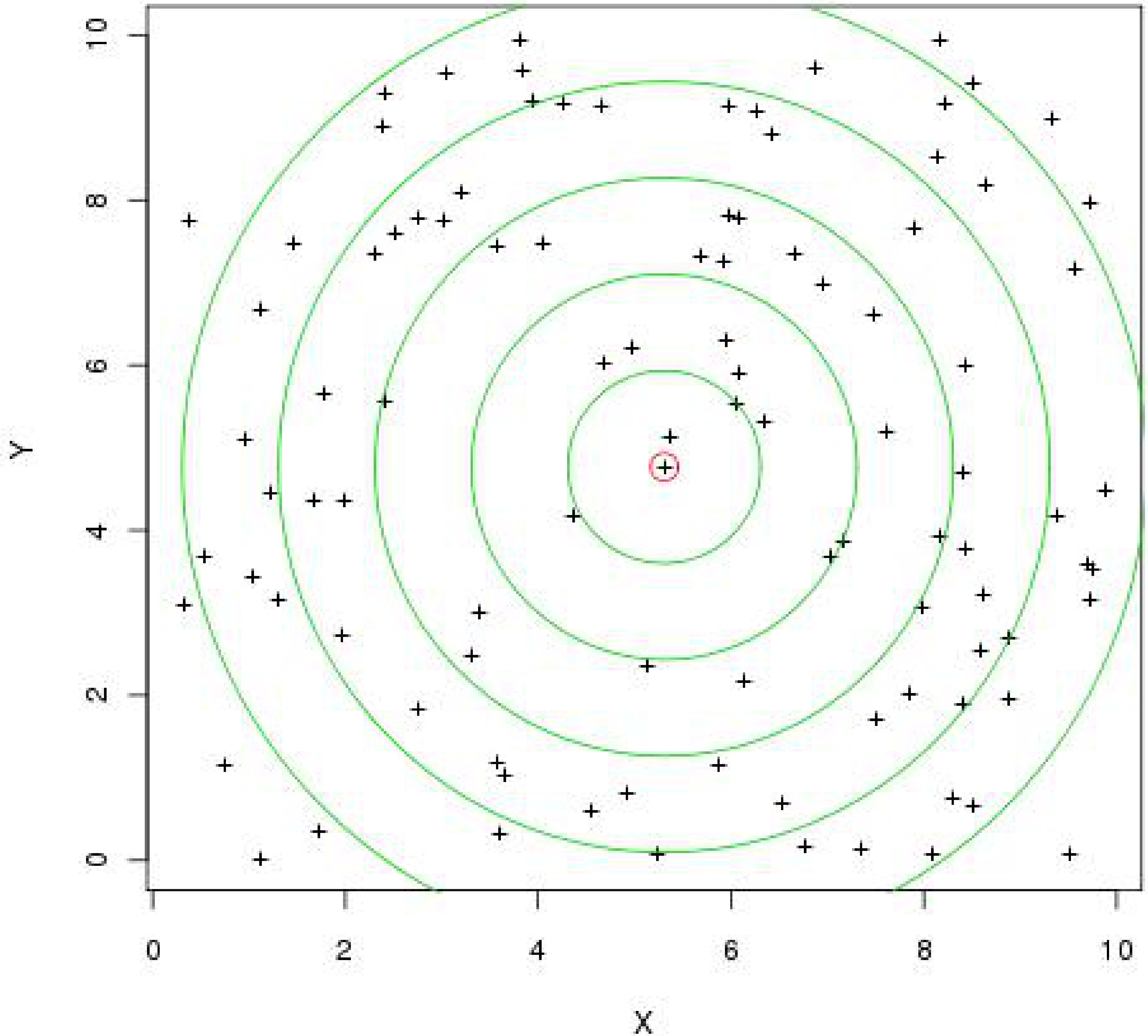
- Interested in the location of points, not their attributes
- Degree of aggregation



Ripley's K

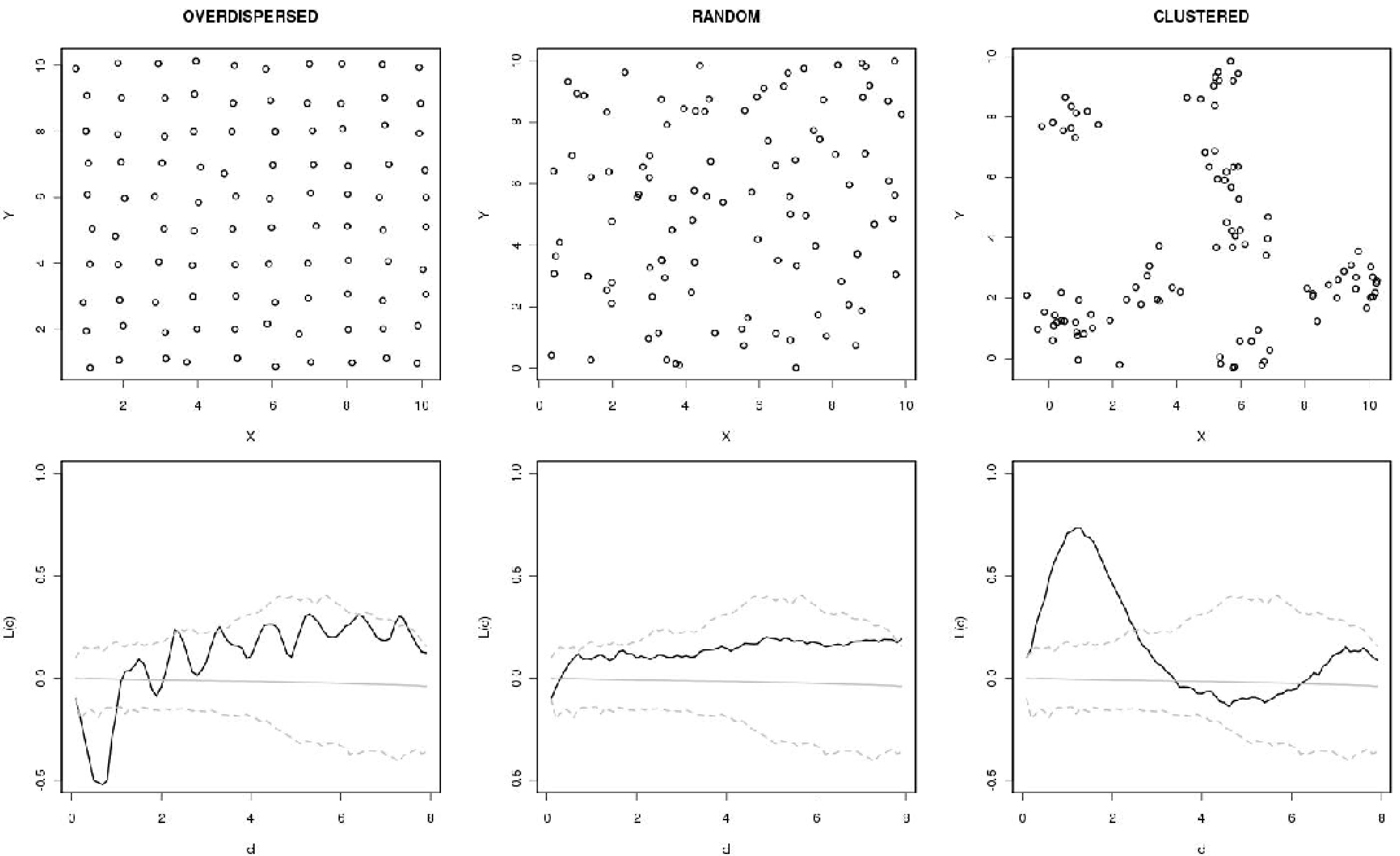
- Calculates counts of points as a function of distance bins for each point
- Combine points together and normalize by area
- Positive = more points expected than random at that distance
- Negative = less than expected
- Intervals by bootstrap
- Requires def'n of area

$$L(d)=\sqrt{\frac{A\sum_{i=1}^{n}\sum_{j=1,j\neq 1}^{n}k(i,j)}{\pi n(n-1)}}$$



Y
X
0
2
4
6
8
10

Ripley's K



Ripley's K in R

```
library(spatial)         ## load library

ppregion(xmin,xmax,ymin,ymax) ## define region

rK <- Kfn(x,max.distance)  ## calculate Ripley's K

plot(rK$x,rK$y-rK$x,type='l',xlab="d",ylab="L(d)")
              ##Plot as L(d) rather than K(d)

## compute and plot interval estimate
Ke <- Kenvl(max.distance, nrep, Psim(n))
lines(Ke$x,Ke$upper-Ke$x,lty=2,col="grey")
lines(Ke$x,Ke$lower-Ke$x,lty=2,col="grey")
```

Applications and Extensions

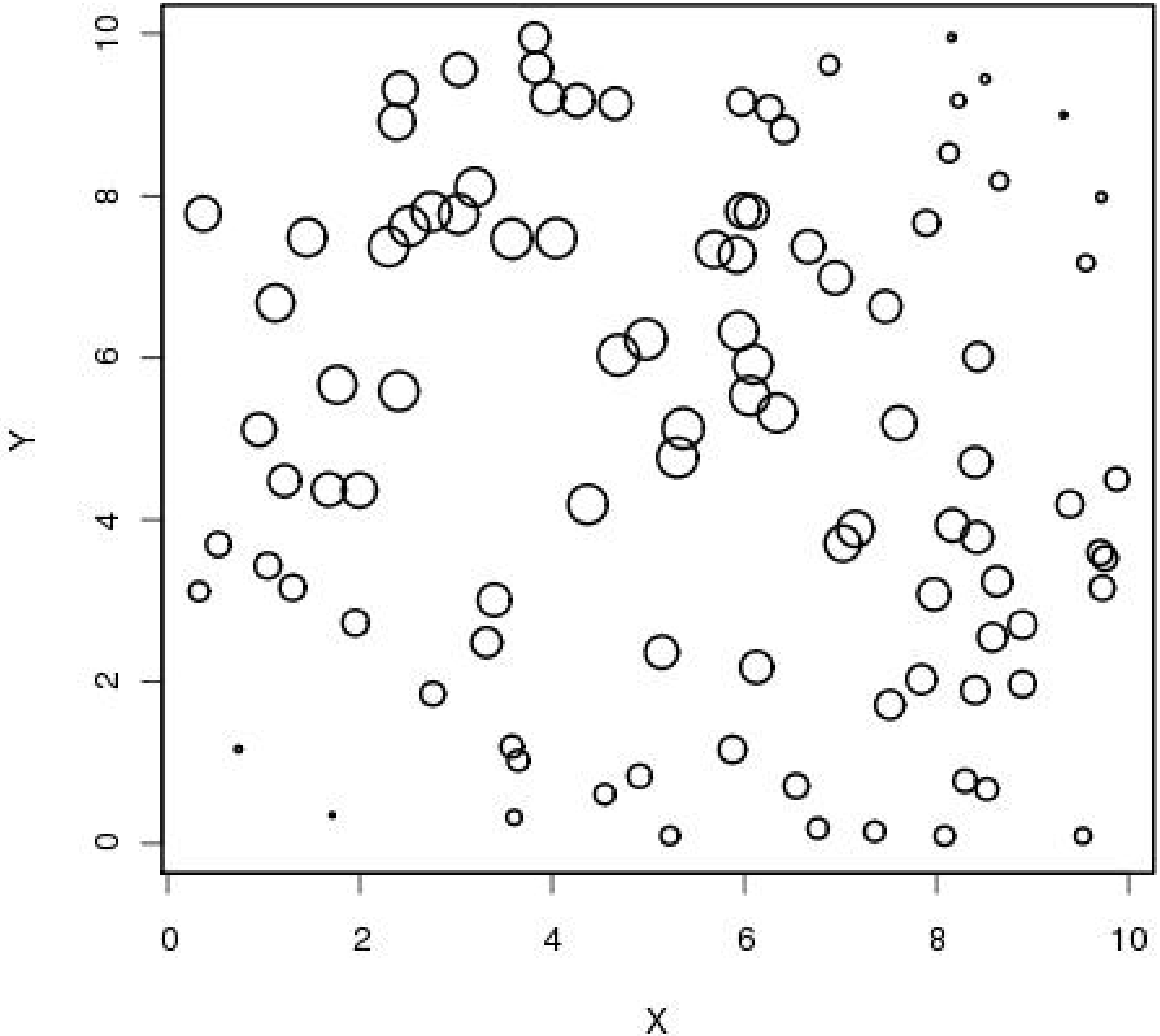
- Irregularly shaped areas
- Choice of points counted in each sum can vary with categorical attribute
- Tree maps
  - Juvenile aggregated (dispersal)
  - Intermediate random (DD mortality)
  - Adults are over-dispersed (crown competition)

Point Referenced Data

- Data has a value/attribute plus spatial coordinates but not area
- Aka geospatial data
  - Origin in mining
- Usually sampling some underlying continuum
- Aims:
  - Account for lack of independence in data due to spatial proximity (analogous to time series)
  - Predict the value at some new location (usually a grid / map)

Examples of Point Ref Data

- Soils
  - Moisture, nutrients, pH, texture, etc.
- Atmospheric or Ocean measurement
  - Surface meteorology (temperature, precip, etc.)
  - CO2, pollutant concentration, salinity, etc.
- Plot data were size of plot << size of domain
  - Biomass/abundance, presence/absence, richness
  - Invasive species, disease prevalence, etc.



0
2
4
6
8
10
X
Y

Geospatial Exploratory Analyses

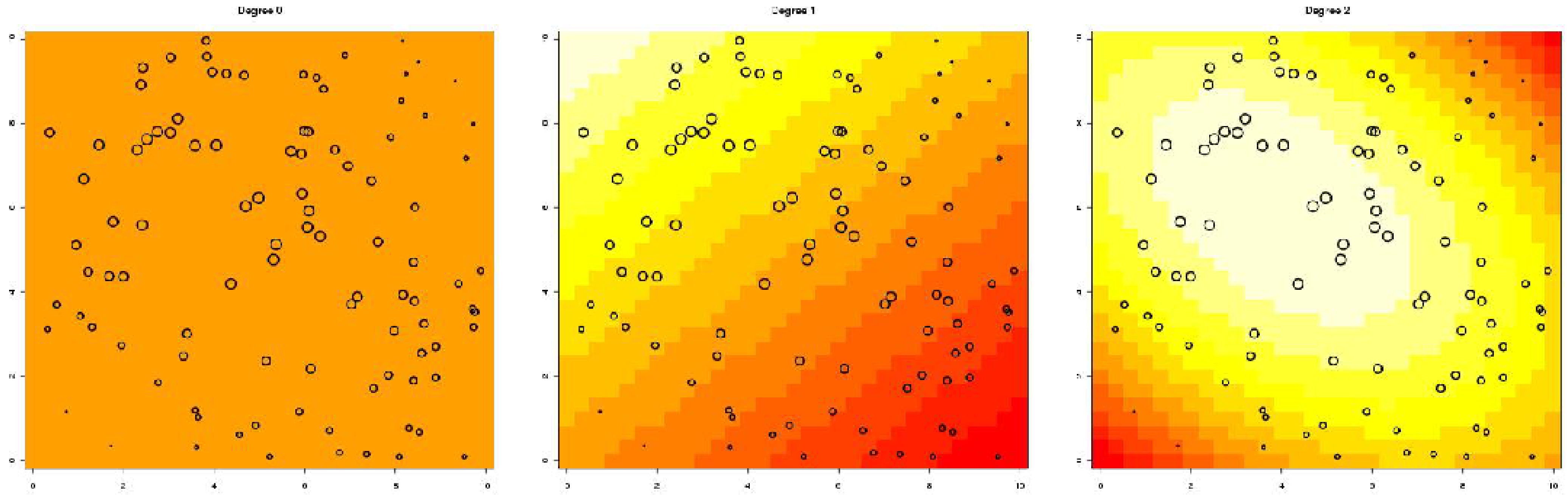
- Smoothing & Detrending
- Autocorrelation
- Interpolation
  - Linear
  - Inverse distance weighed
  - Geostatistical (Kriging)

- Many packages in R, will focus on most basic & "built in"

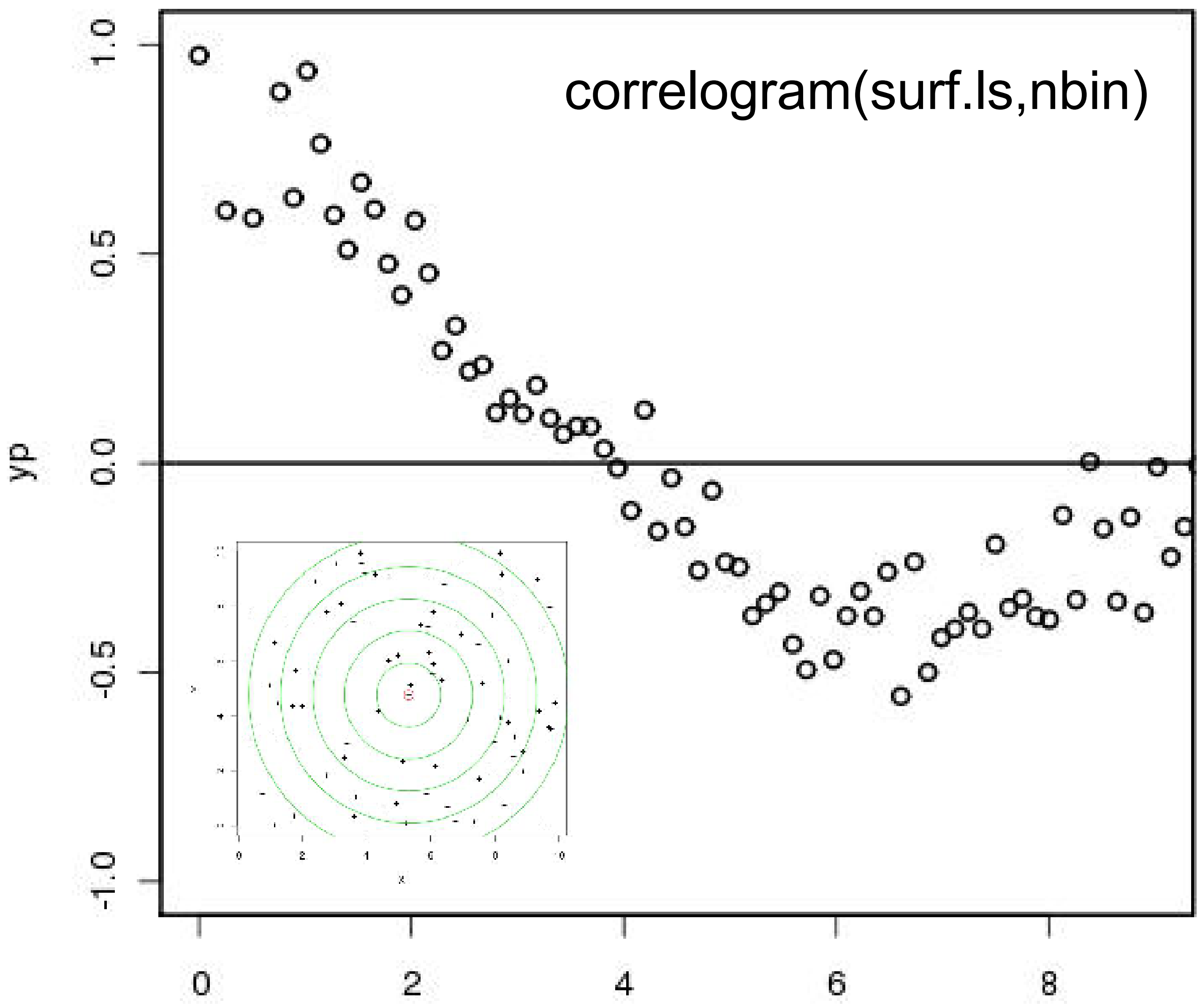
Smoothing / Detrending

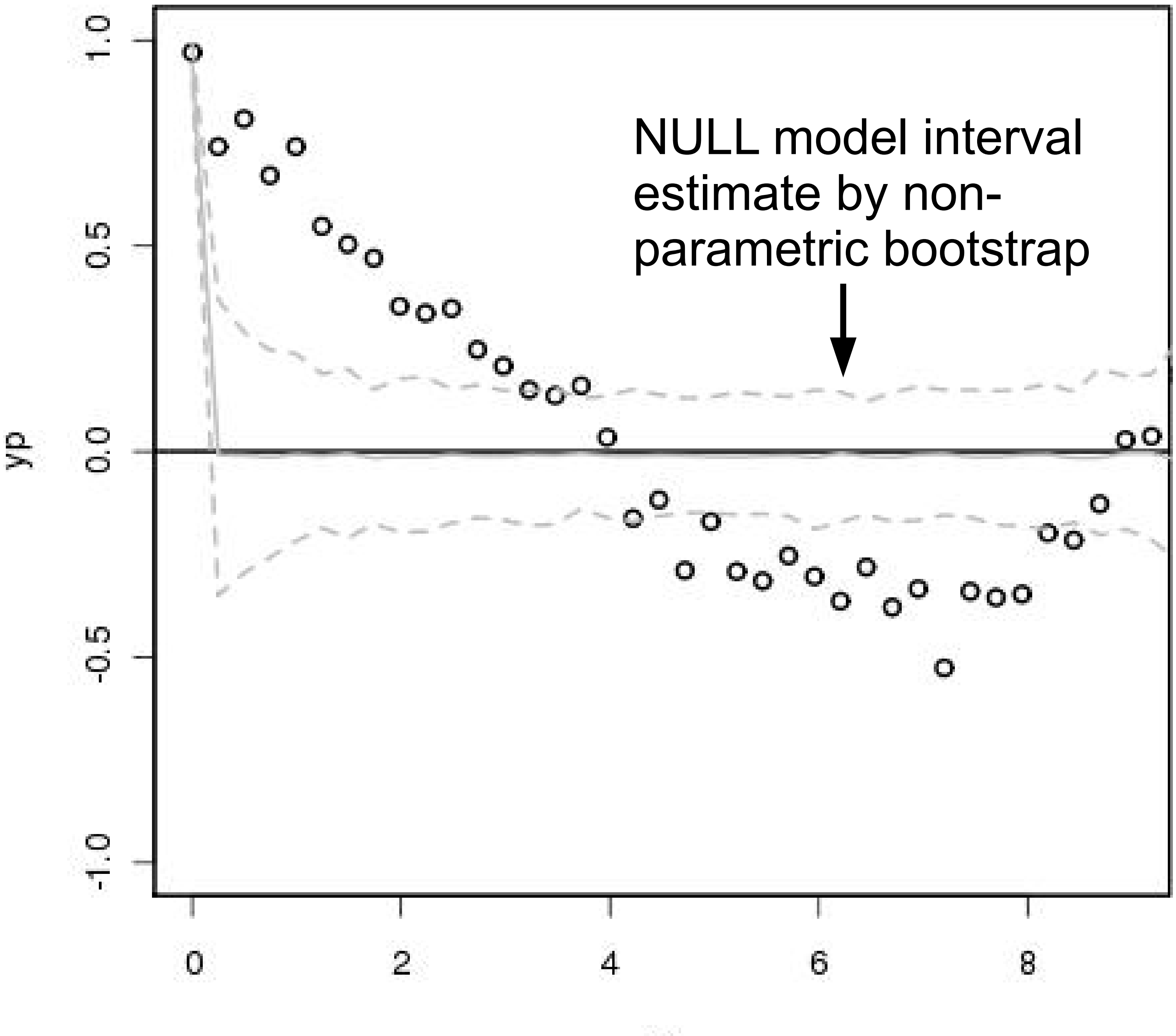
- **Objective**: Like with time-series, most statistical methods assume **stationarity**
- More complicated in 2D (sparse, irregular)
- Polynomial (in R, library(spatial))
  - Fit surface: **surf.ls**(degree, x, y, z)
  - Project: **trmat**(surf.obj, xmin, xmax, ymin, ymax, n)
  - Plot: **image**(tr.obj)

Degree 0 Degree 1 Degree 2



Spatial autocorrelation



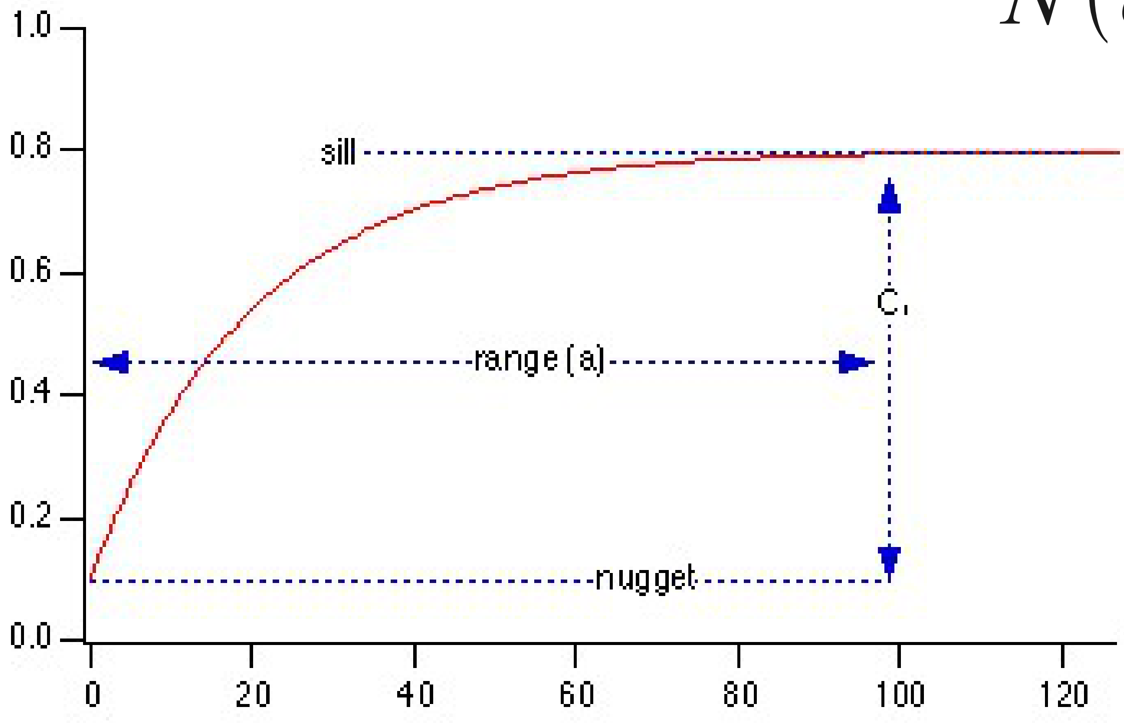


NULL model interval estimate by non-parametric bootstrap
yp
xp
1.0
0.5
0.0
-0.5
-1.0
0
2
4
6
8

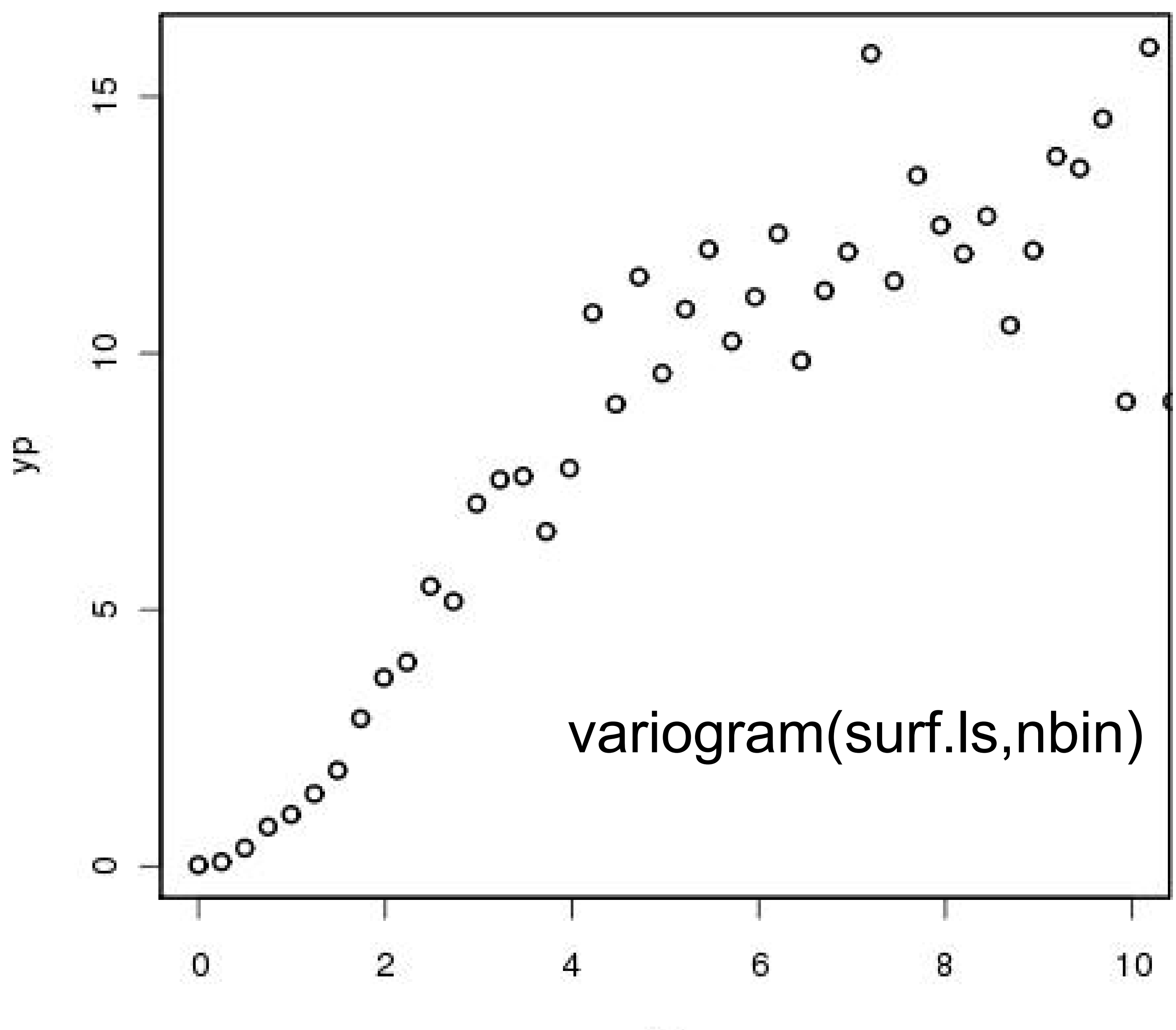
Variogram

- Traditionally, autocorrelation in geostatistics has been expressed in terms of a **variogram** or semivariogram
- Units = variance

$$\gamma(d) = \frac{1}{N(d)} \sum_{i,j \in d}^{N(d)} (Z_i - Z_j)^2$$



- Sill = asymptote
- Range = distance to asymptote
- Nugget = variance at lag 0


variogram(surf.ls,nbin)
yp
xp
0
5
10
15
0
2
4
6
8
10

Spatial Covariance

- If C(d) is the spatial covariance

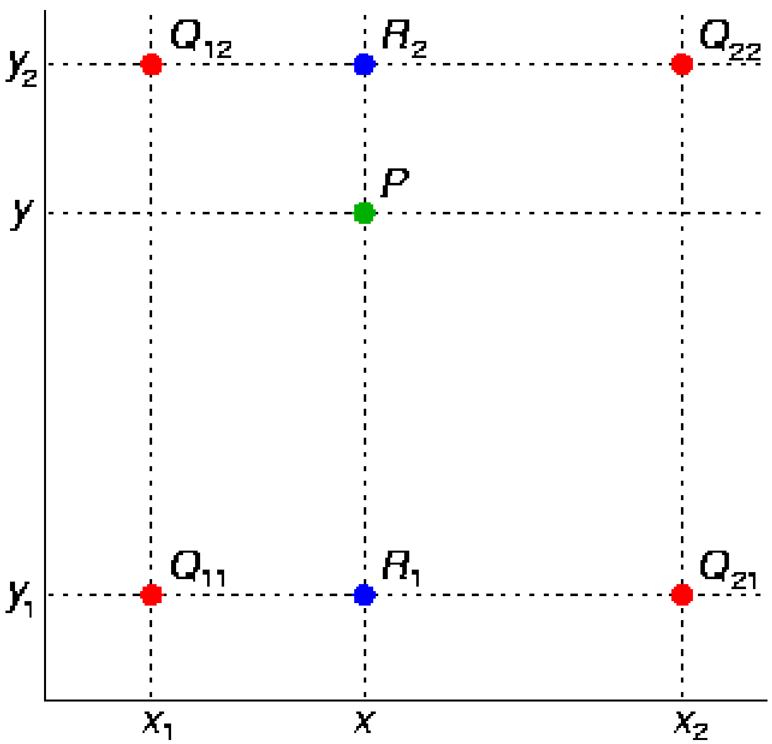
$$C(d) = COV[Z_x, Z_{x+d}]$$

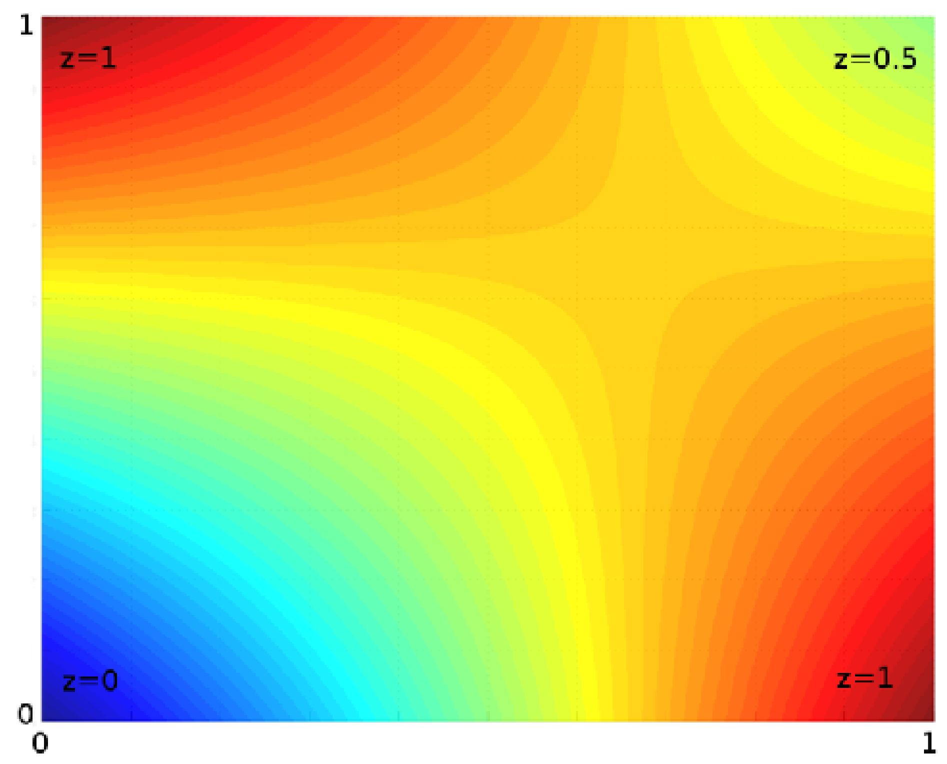
- Autocorrelation : $\rho(d) = C(d)/C(0)$

- Variogram : $\gamma(d) = C(0) - C(d)$

Interpolation

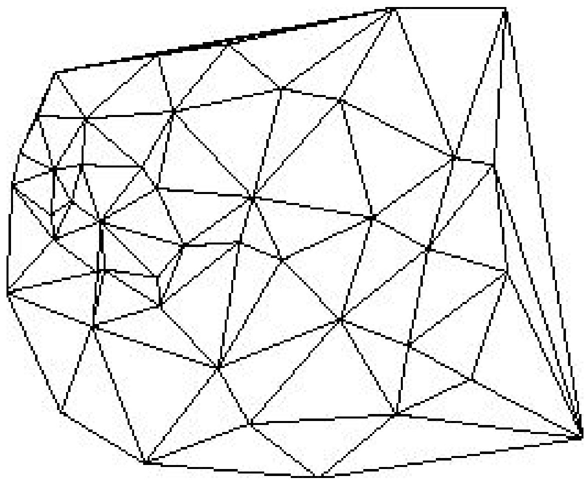
- Objective: predict Z at some new point(s)
  - Often on a grid to make a raster map
- Linear
  - Simplest if data already on a grid (four corners)

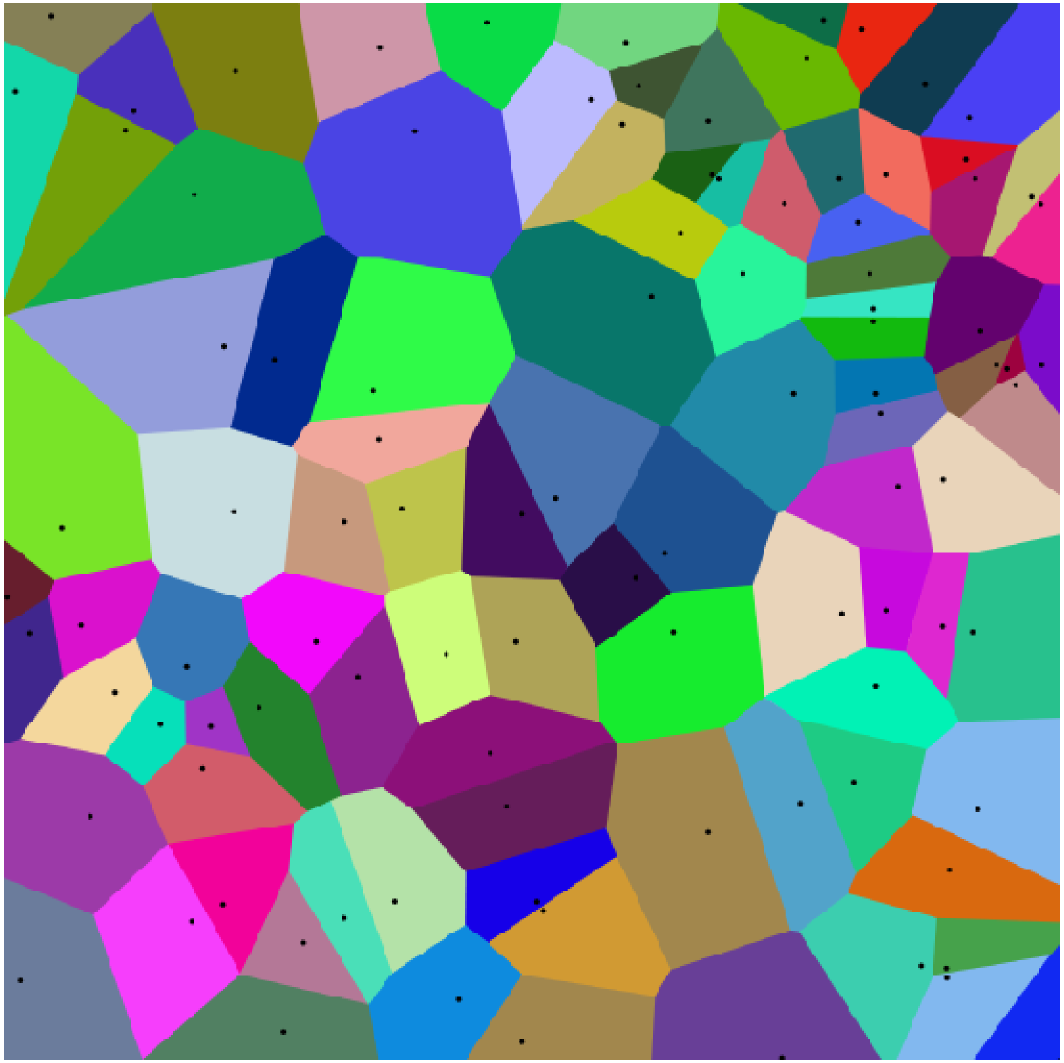




Interpolation

- Bicubic interpolation: cubic analog to bilinear
- Nearest-Neighbor:
  - Tesselation
  - Voronoi Diagram
- Triangular irregular network (TIN)





Inverse-Distance Weighted

- Previous methods only used nearest points
- All are special cases of a weighted average
- For irregular, often want to use n-nearest points or a fixed search radius (variable number of points)
- Requires a way of WEIGHTING points as a function of distance
- Inverse-distance weighted: $W_{ij} = 1/d_{ij}$
- $Z_i = \Sigma W_{ij} Z_j / \Sigma W_{ij}$

Spatial Weighted Averages

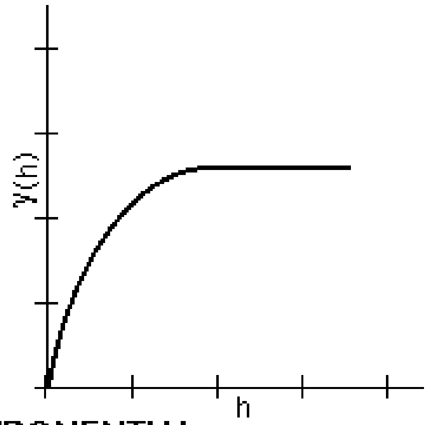
- Other alternatives to 1/d (e.g. $1/d^2$)
- Major criticisms
  - Choice of weighting function somewhat arbitrary, not connected to properties of the data
  - Does not account for error in interpolation
    - Points further from known points should be more uncertain
- Interpolation vs smoothing
  - Interpolation always passes exactly though the data points (0 residuals)
  - Smoothing separates trends + residuals

Kriging

- Interpolation based on autocorrelation fcn
- Requires fitting an autocorrelation model to the variogram or correlogram
  - Provides “weight” to points based on observed relationship between distance and correlation
  - Requires choice of parametric function
- Provides mechanism for estimating interpolation error

Variogram Models

SPHERICAL

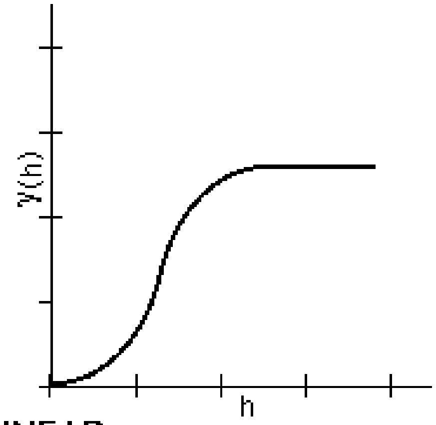


$$\gamma(\mathbf{h}) = c_0 + c\left(\frac{3h}{2a} - \frac{1}{2}\left(\frac{h}{a}\right)^3\right) \qquad 0 < h \le a$$

$$\gamma(\mathbf{h}) = c_0 + c \qquad h > a$$

$$\gamma(0) = 0$$

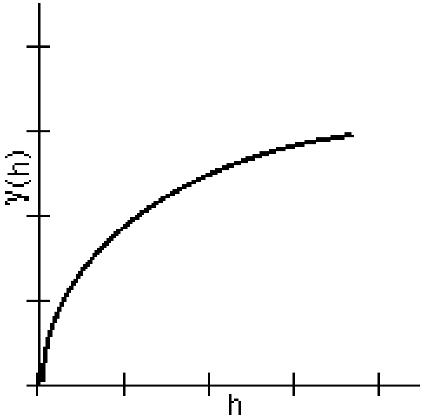
GAUSSIAN



$$\gamma(\mathbf{h}) = c_0 + c\left(1 - exp\left(-\frac{h^2}{r^2}\right)\right) \qquad h > 0$$

$$\gamma(0) = 0$$

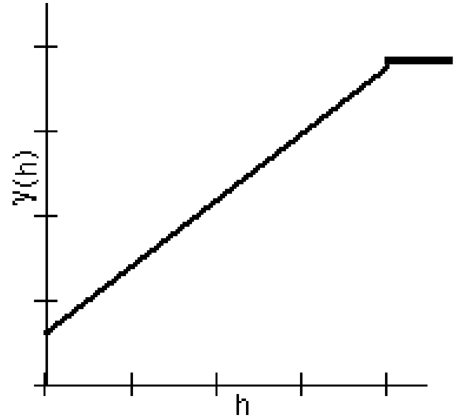
EXPONENTIAL



$$\gamma(\mathbf{h}) = c_0 + c\left(1 - exp\left(\frac{-h}{r}\right)\right) \qquad h > 0$$

$$\gamma(0) = 0$$

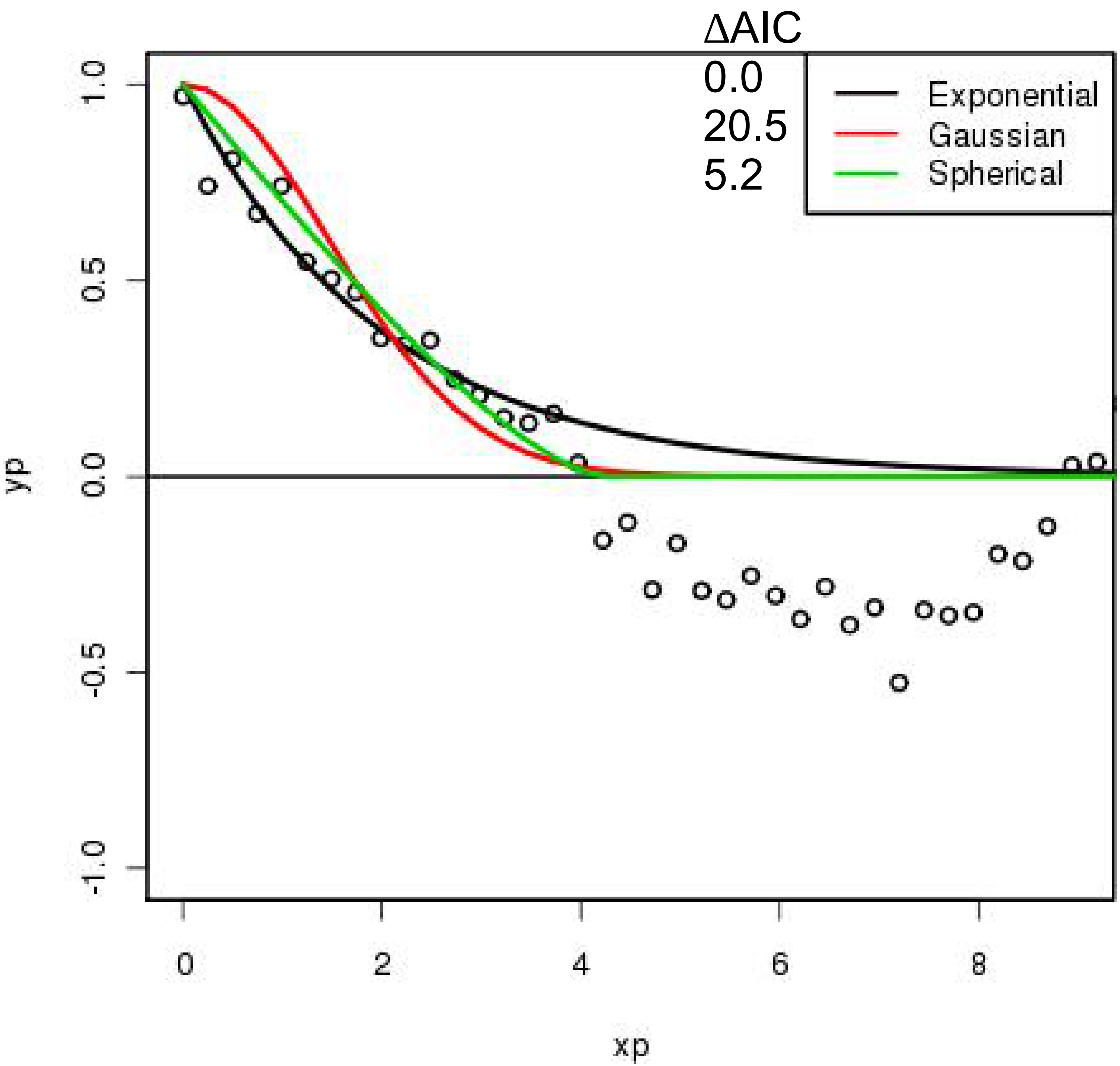
LINEAR

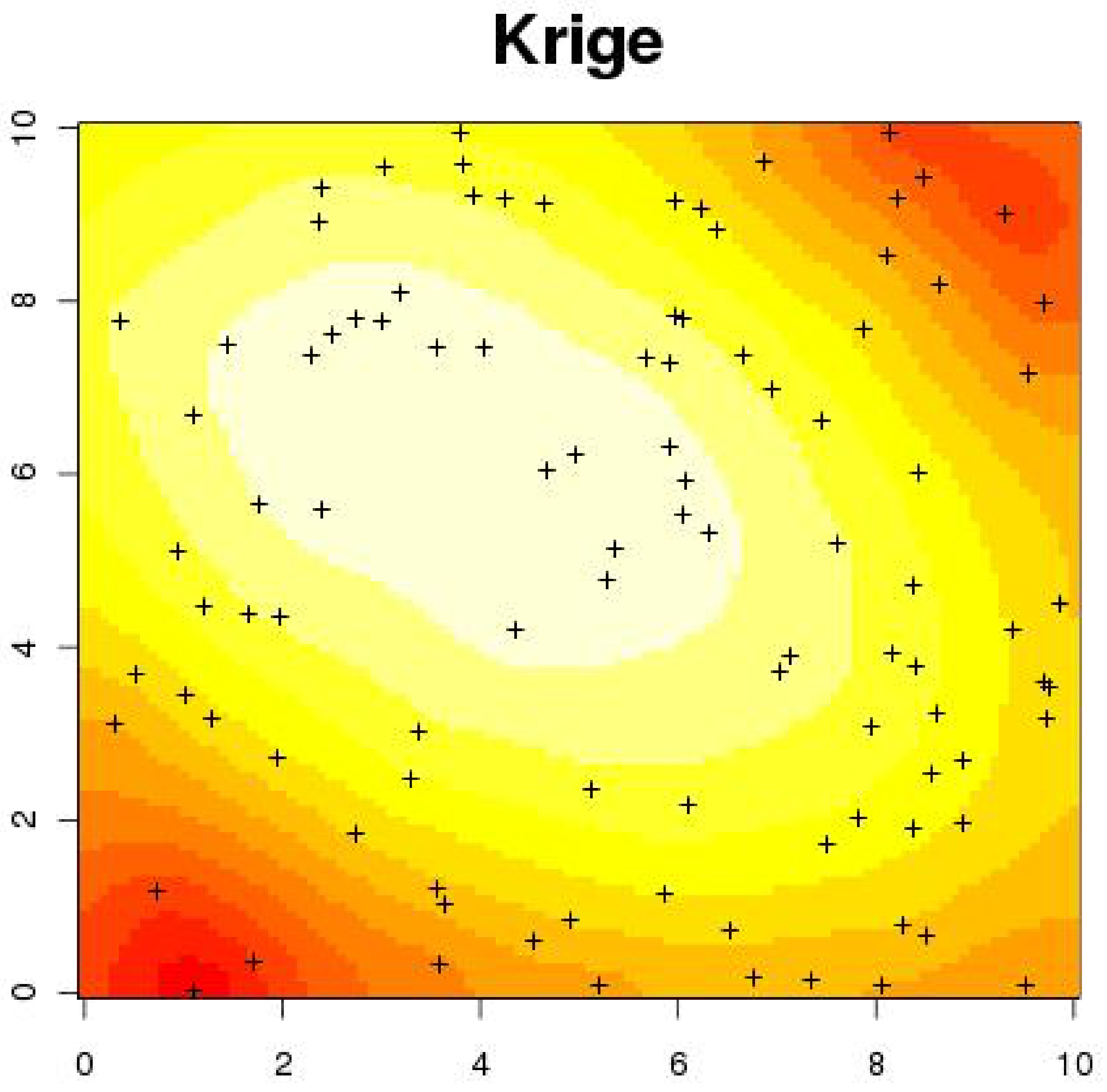


$$\gamma(\mathbf{h}) = c_0 + c\left(\frac{h}{a}\right) \qquad 0 < h \le a$$

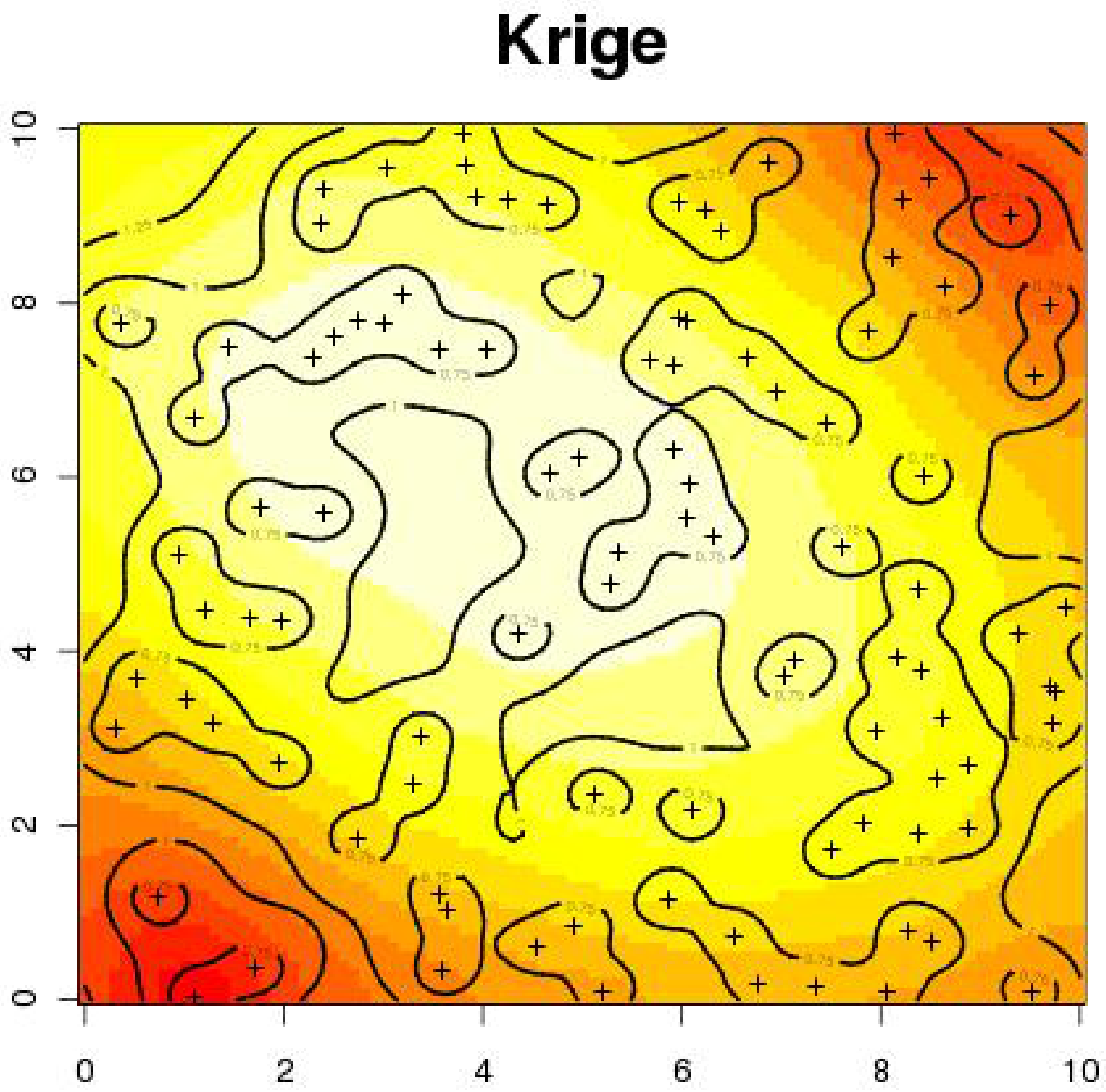
$$\gamma(\mathbf{h}) = c_0 + c \qquad h > a$$

$$\gamma(0) = 0$$


ΔAIC
0.0
20.5
5.2
Exponential
Gaussian
Spherical
1.0
0.5
0.0
-0.5
-1.0
yp
0
2
4
6
8
xp



Krige
0
2
4
6
8
10
0
2
4
6
8
10


Krige
0
2
4
6
8
10
0
2
4
6
8
10

Step 1: Fit variance model

```
##correlogram
cg <- correlogram(data,nbin)

##fit covariance function
expfit <- function(parm){
   -sum(dnorm(cg$y,
         expcov(cg$x,parm[1]),
         parm[2],log=TRUE))
}
efit <- optim(ic,expfit)
```

Built in function for exponential covariance

$$\prod N(y|f(x|\alpha),\sigma^2)$$

↓

$$-\sum \log\left(N(y|f(x|\alpha),\sigma^2)\right)$$

Step 2: Krige surface

```
##detrend accounting for covariance
kr <- surf.gls(degree,expcov,data,d=efit$par[1],...)

## matrix prediction (Kriging)
pr <- prmat(kr, xmin, xmax, ymin, ymax, n)
image(pr)

## matrix error
se <- semat(kr, xmin, xmax, ymin, ymax, n)
contour(se3,add=TRUE)
```

Anisotropy

- In addition to STATIONARITY (spatial covariance is the same at all locations), spatial models also assume ISOTROPY, that the spatial covariance is the same in all DIRECTIONS
- Calculate/fit variogram separately for different directions (angular bins) to account for anisotropy
  - Increases # of parameters, less data points as bins get smaller
  - Alt: modify cov fcn to account for direction
  - Alt: fit cov fcn to different subdomains (location)

Flavors of Kriging

- Simple Kriging: mean = 0
- Ordinary Kriging: mean = unknown μ
- Universial Kriging: mean = polynomial trend
- Cokriging: inclusion of covariates

Limitations of Kriging

- Assumes the variogram model is known
  - Dropped parameter error
- Fitting of variogram model:
  - Not done as part of overall model fit
  - Not done on data directly
    - Binned means of all n^2 pairwise differences
- Detrending and autocorr done separately
- Sometimes just want non-independence
- Similar to T.S., OK for EDA but ultimately want to fit whole model at once.