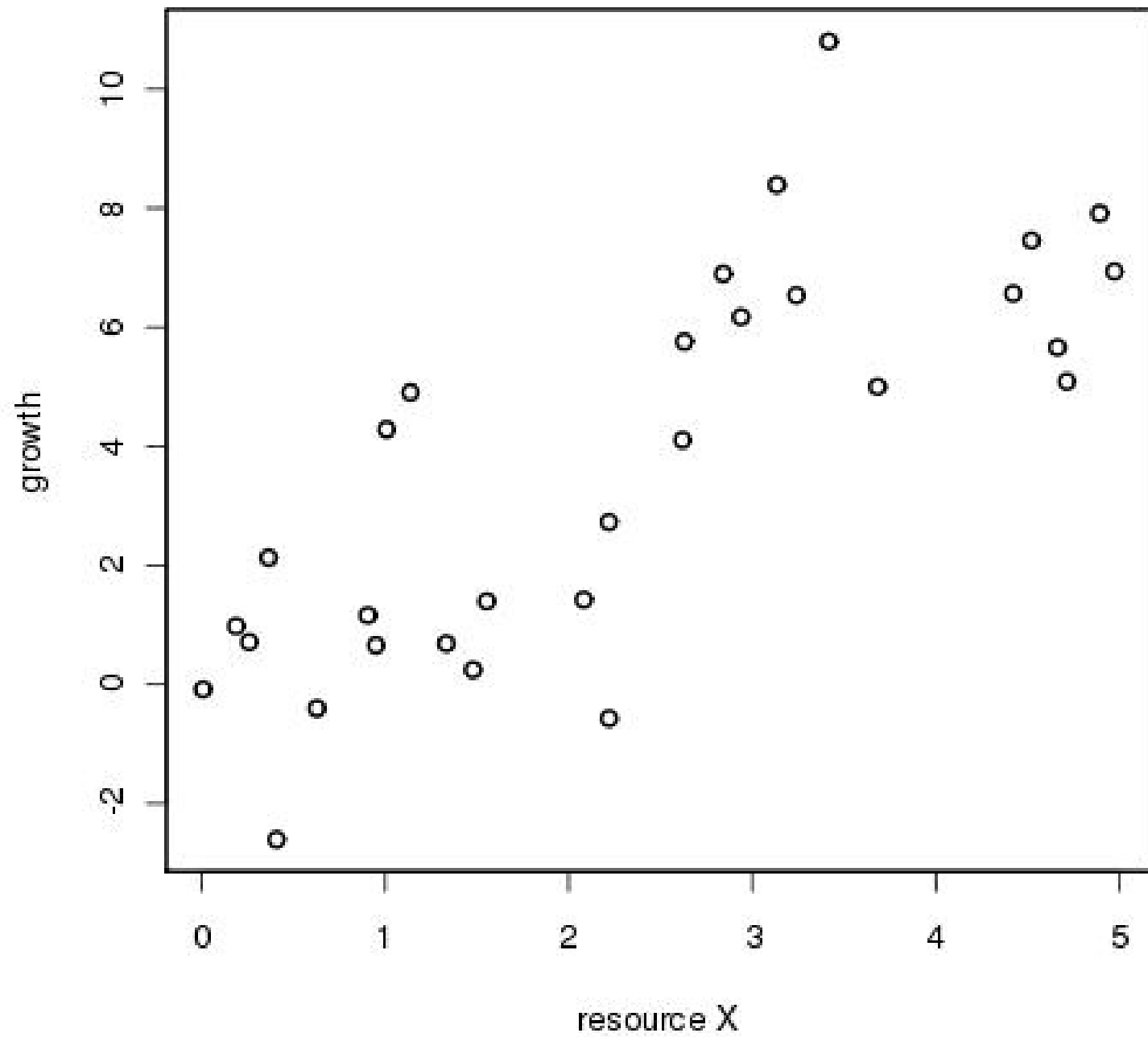# Repeated Measures and Interventions

Longitudinal Data
(aka Repeated Measures)

- The same observational unit (plot, individual, etc) is often measured repeatedly over time

- Usually have <u>many</u> such observation units

- Observations on the same unit over time are not independent

Alternatives for Repeated Measures

- ## Random effects
  - By time: Assumes all observational units move up or down in sync
    - probably won't solve lack of independence
  - By unit: Assumes a unit is offset from "average" by some constant amount
- ## Autoregressive: AR(1)
  - Assumes each unit is similar from one time step to the next but not that units are synchronized
- ## With short t.s. almost impossible to distinguish AR vs individual effects
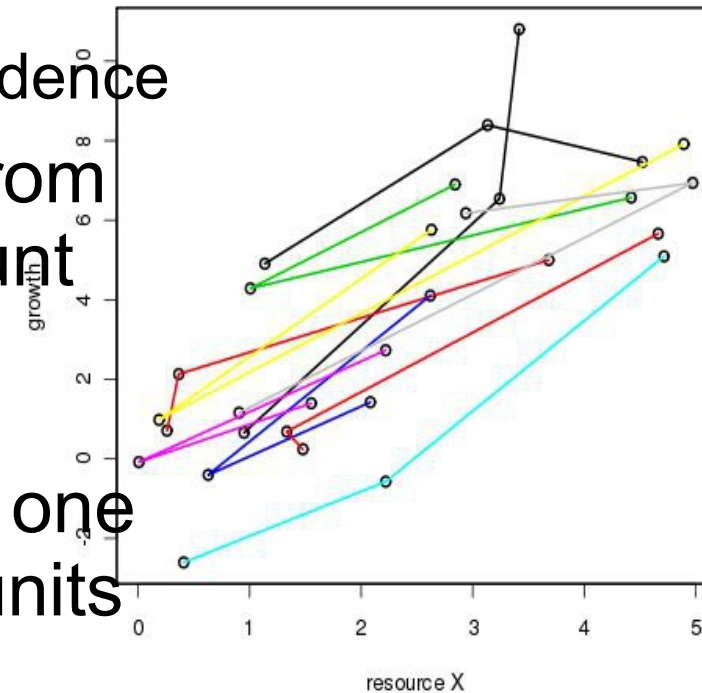
Example: Growth

- Consider a population of individuals censused 3 times who's growth is a function of X

$$\mu_{i,t} = \beta_0 + \beta_1 X_{i,t}$$

- If we assume a common variance and autocor.

$$\vec{g}_i \sim N_3(\vec{\mu}_i, \Sigma)$$

$$\Sigma = \frac{\sigma^2}{1-\rho^2} \begin{vmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & \rho \\ \rho^2 & \rho & 1 \end{vmatrix}$$
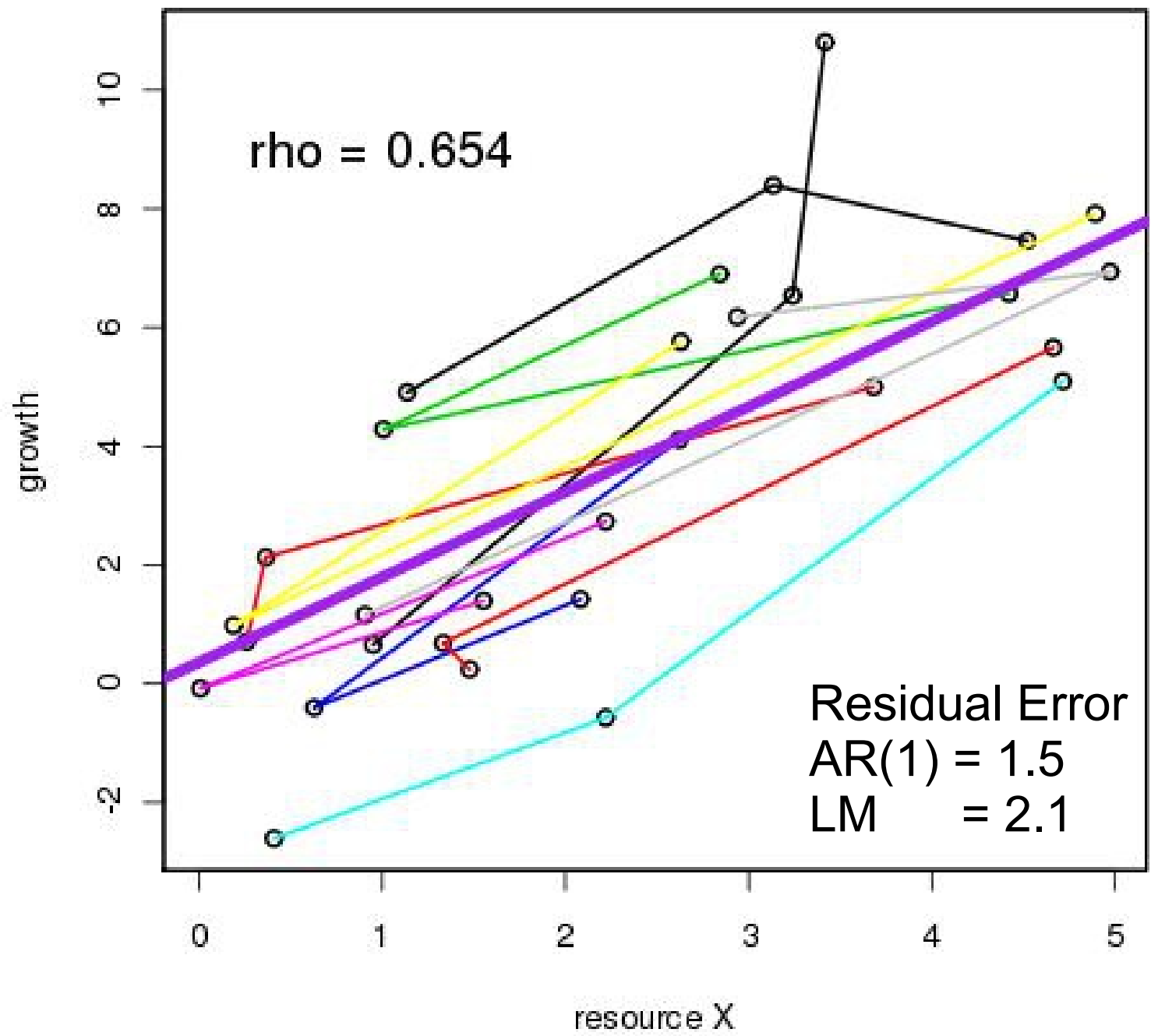
Likelihood Approach

```
library(mvtnorm)
H <- matrix(c(0,1,2,1,0,1,2,1,0),3,3)

lnlik <- function(theta){
  beta  <- theta[1:2]
  sigma <- theta[3]
  rho   <- theta[4]
  SIGMA <- sigma/(1-rho^2)*rho^H

  L = 0
  for(i in 1:n){
    L = L - dmvnorm(g[i,],beta[1]+beta[2]*X[i,],SIGMA,log=TRUE)
  }

  return(L)

}
```

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & \rho \\ \rho^2 & \rho & 1 \end{bmatrix}$$

rho = 0.654

Residual Error
AR(1) = 1.5
LM   = 2.1

growth

resource X

Bayesian Approach

```
model{
  beta ~ dmnorm(b0,Vb)
  sigma ~ dgamma(0.01,0.01)
  rho   ~ dunif(-1,1)
  SIGMA <- inverse(1/sigma/(1-rho^2)*rho^H)

  for(i in 1:n){
    mu[i,] <- beta[1]+beta[2]*X[i,]
    g[i,] ~ dmnorm(mu[i,],SIGMA)
  }
}
```

State Space approach

- Bayesian version of AR(1) repeated measures did not separate process and measurement uncertainty

- Easy to extend the State Space model to the repeated measures context with a common rho

$$g_{i,t} \sim N(\beta_0 + \beta_1 X_{i,t} + \rho\, g_{i,t-1}, \sigma^2)$$

$$g_{i,t}^{(o)} \sim N(g_{i,t}, \tau^2)$$
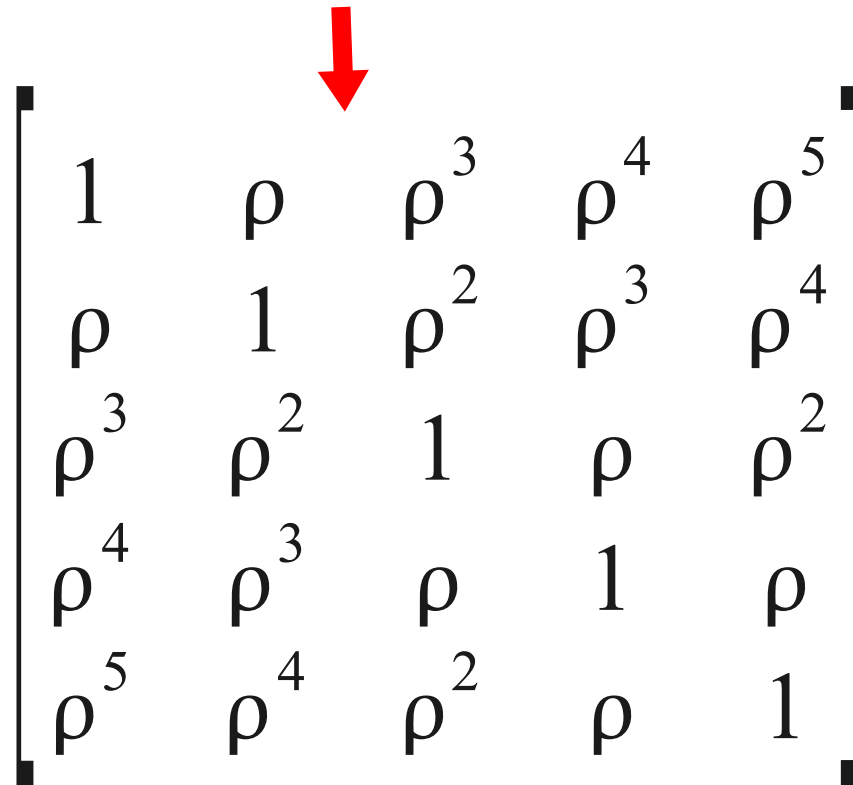
Generalization of Repeated Measures

- Can accommodate mixed models / random effects (though some would be redundant)

- Can accommodate nonlinear models

- Covariance matrix ONLY works with NORMAL PDF

- State Space version can work with most any PDF because we separate the data model (not autocorrelated) from the process error (autocorrelated)

Missing Data Covariance Matrix

- Drops missing ROWS and COLUMNS from matrix

Missing 3rd observation so drop both the third row and third column from the full matrix

[X1 X2 X4 X5 X6]

$$\begin{bmatrix} 1 & \rho & \rho^3 & \rho^4 & \rho^5 \\ \rho & 1 & \rho^2 & \rho^3 & \rho^4 \\ \rho^3 & \rho^2 & 1 & \rho & \rho^2 \\ \rho^4 & \rho^3 & \rho & 1 & \rho \\ \rho^5 & \rho^4 & \rho^2 & \rho & 1 \end{bmatrix}$$

Hierarchical AR models

- Thus far have focused on autocorrelation in the STATE variables as part of the process model

- In hierarchical models, can also model autocorrelation in the hierarchical parameters

    - Time-varying model parameters

$$\vec{\alpha} \sim N(0, \tau^2)$$

$$\downarrow$$

$$\vec{\alpha} \sim N(0, \mathbf{T})$$
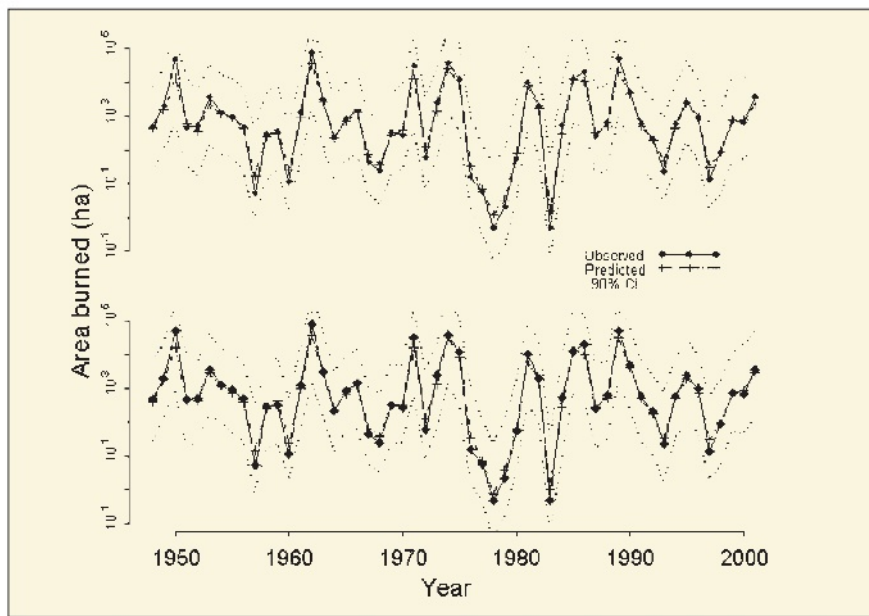
Fire in the Florida Everglades

Figure 2. Predicted (median) and observed area burned in the April–May wildfire season in Everglades National Park, 1948–2001. (top) One-year predictions. (bottom) Three-month predictions that included mean November–January SOI as a covariate.

ARMA(2,2) with temporally varying parameters
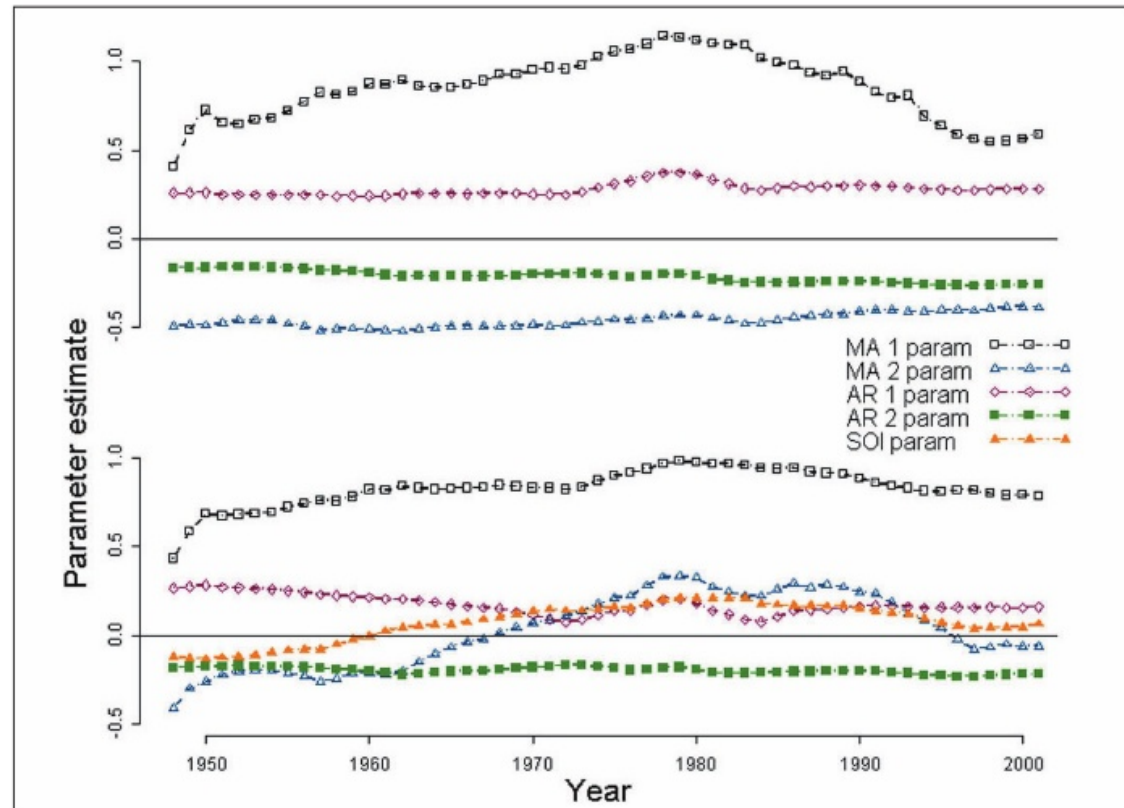
Beckage and Platt 2003

Figure 3. Evolution of model parameters (median values) through time. (top) One-year predictions. (bottom) Three-month predictions that included mean November–January SOI as a covariate.

Intervention Analysis

- Treatment effects in TIME

  – Pretreatment data establishes unit differences

- Hypotheses are usually that one or more parameters changed with treatment

- Alternate model (NULL) is no change in parameters with treatment

- Time can be modeled as

  – Covariate (explicit)

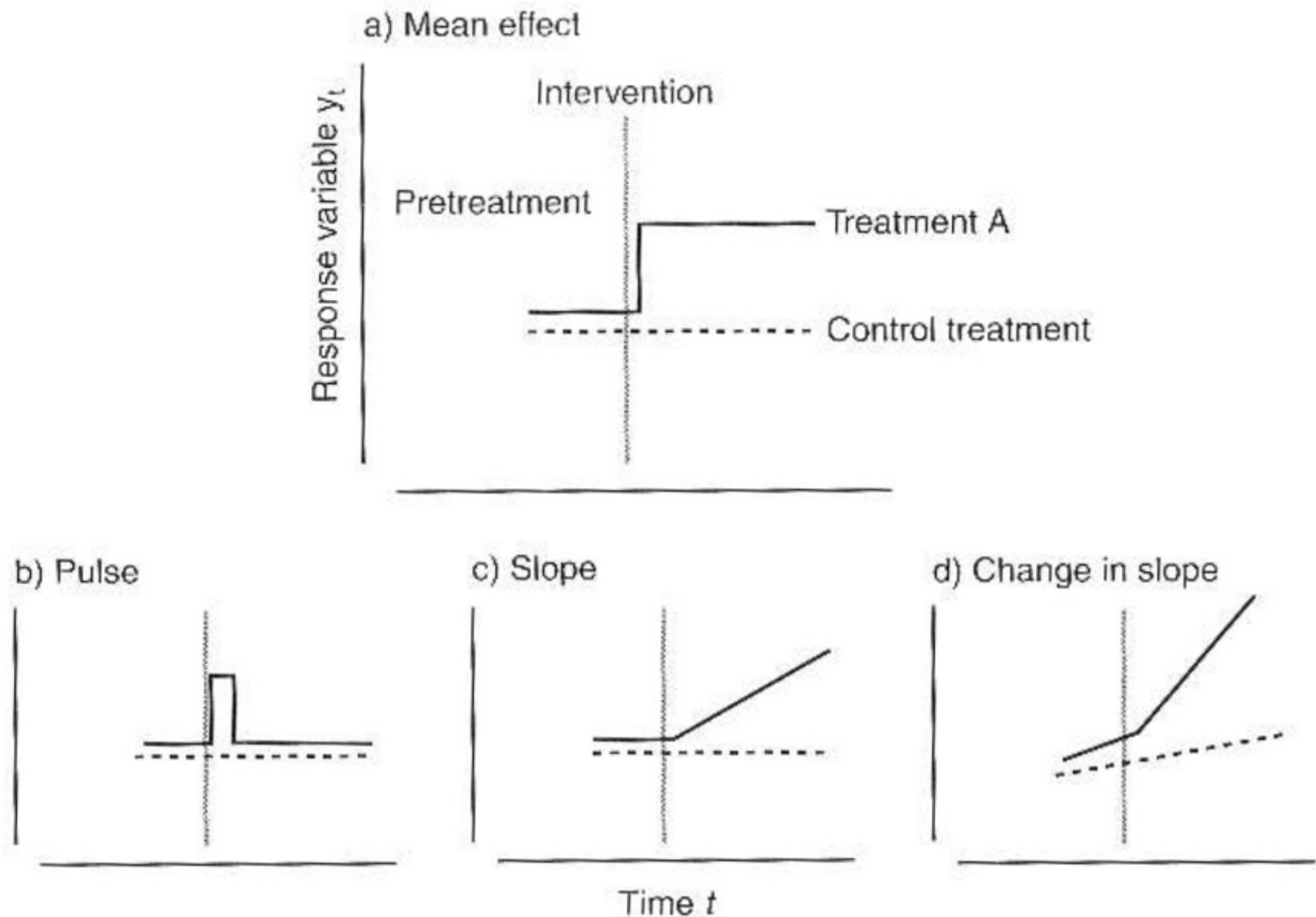 – Time varying treatment (implicit)

  – Autocorrelation

FIGURE 9.21. Four examples of ways in which an intervention might occur. The trend in the solid line "Treatment A" might not match that of the response to the treatment. For example, a step change in a treatment x (a) might elicit a trajectory of change in a response variable y.

Change point / threshold detection models

- Like Intervention models but the point in time where the change in parameters occurs is itself an unknown that needs to be estimated

- Challenging for likelihood models because the likelihood has a discontinuity

```
beta1 ~ dmnorm(b0,Vb)  ## prior betas
beta2 ~ dmnorm(b0, Vb)
prec ~ dgamma(s1,s2)  ## prior precision
K ~ dcat(pi)      ##discrete prior for breakpoint

for (i in 1:n){
  mu1[i] <- beta1[1] + beta1[2]*time[i] ##prebreak
  mu2[i] <- beta2[1] + beta2[2]*time[i] ##postbreak
  mu[i] <- ifelse(i>K,mu2[i],mu1[i])   ##process model
  y[i]  ~ dnorm(mu[i],prec)    ## data model
}
```