Debt Dynamics and Default Probabilities

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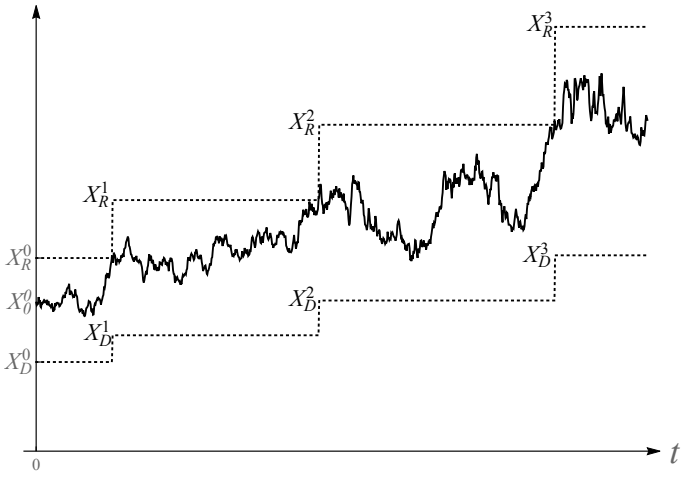
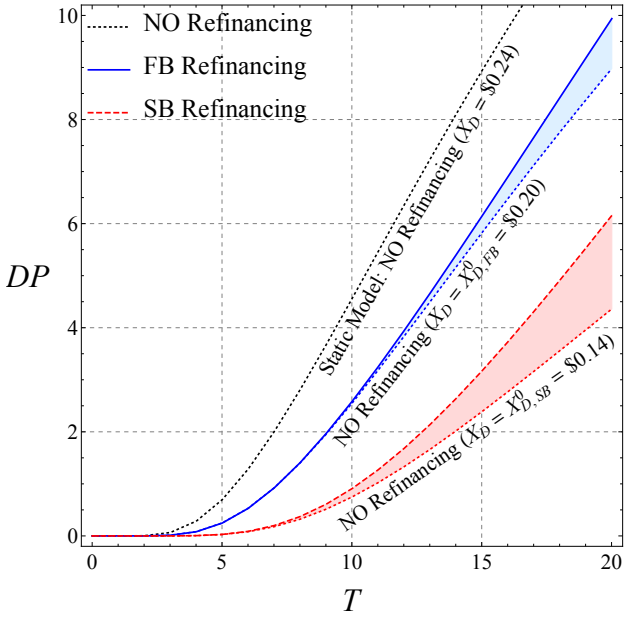
Financial analysts and researchers have been interested in predicting corporate default probabilities (*DP*s) for many years. While *DP* modeling is well-understood and widely-used (see, e.g., 9th generation of the Moody's Analytics Public Firm Expected Default Frequency model), the assumption that firms borrow only once and keep a constant debt amount during the time horizon for *DP* predictions is unrealistic, especially for longer horizon *DP*s. In practice, firms adjust their leverage optimally over time. Therefore, *DP*s not only need to reflect the dynamics of a firm's earnings over time but also its debt dynamics.

In recent years, a class of dynamic models emerged, in which a firm infrequently but optimally restructures its debt by returning to a target leverage ratio. Given earnings $X_0 = \$1$ at time 0, earnings growth and volatility, the firm balances interest tax shields against bankruptcy and issuance costs of debt. This implies (i) the default barrier, X_D^i, changes for each refinancing stage i and (ii) an upper refinancing barrier X_R^i at which the firm moves from stage i to stage $i+1$. The lower barrier maximizes equity value. For the upper barrier, "FB Refinancing" assumes commitment at time 0 to firm value-maximization (first-best), while "SB Refinancing" considers equity value-maximization without commitment (second-best). The figure charts changing double-barriers due to debt dynamics and earnings dynamics over time t.



For "FB Refinancing" and realistic parameters, initially optimal leverage is 37%. At the default barrier, leverage rises to 100%. At the refinancing barrier, leverage declines to about 16% prior to returning to the optimal leverage target of 37%.

Notably, we derive an analytic solution for *DP* estimates over a fixed time horizon T of a company that makes coupon payments on its debt and infrequently returns to its leverage target by increasing its debt unless it defaults on its debt. In the figure, the dotted "NO Refinancing" lines represent the conventional *DP*s for three default barriers (i.e., correct one from static model and incorrect or naive ones from dynamic models), while the blue, solid and red, dashed lines depict the new solutions. The shaded area between the lines is the effect of debt dynamics on *DP*s.



Compared to the conventional *DP* formula for a constant default barrier, our changing-barrier *DP* formula recognizes that the barrier changes dynamically (i.e., is ratcheted up over time). Our results have several implications that improve *DP* estimates.

- The conventional solution's underestimation error is up to 11% for first-best policies (41% for second-best policies). Underestimation errors vary with parameter values too.
- Under second-best policies, the influence of debt dynamics on *DP*s is larger, even though optimal leverage is lower. The reason is that debt restructuring is more frequent.
- The dynamic model lowers optimal leverage from 44% for "NO Refinancing" to 37% (27%) for "FB Refinancing" ("SB Refinancing"). Thus, for an otherwise identical firm's leverage from a static model, *DP*s are considerably higher.

To summarize, it is crucial to consider a firm's debt dynamics when estimating its *DP*s, especially over longer time horizons.