Online Appendices to Financing Asset Sales and Business Cycles

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August 22, 2017

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Appendix A. Derivations

Appendix A.1. The stochastic discount factor, risk-free rates, and market prices of risk

According to Bhamra, Kuehn, and Strebulaev (2010) and Chen (2010), solving the Bellman equation associated with the consumption problem of the representative agent implies that the stochastic discount factor m_t follows the dynamics

$$\frac{\mathrm{d}m_t}{m_t} = -r_i \mathrm{d}t - \eta_i \mathrm{d}W_t^C + \left(e^{\kappa_i} - 1\right) \mathrm{d}M_t, \tag{A.1}$$

in which M_t is the compensated process associated with the Markov chain. r_i are the regime-dependent risk-free interest rates, η_i the risk prices for systematic Brownian shocks affecting aggregate consumption, and κ_i the market prices of jump risk:

$$r_i = \bar{r}_i + \lambda_i \left[\frac{\omega - \delta}{\omega - 1} \left(w^{-\frac{\omega-1}{\omega-\delta}} - 1 \right) - \left(w^{-1} - 1 \right) \right], \tag{A.2}$$

$$\eta_i = \omega \sigma_i^C, \tag{A.3}$$

$$\kappa_i = (\delta - \omega) \log \left(\frac{h_j}{h_i} \right), \tag{A.4}$$

with $i, j = G, B, i \neq j$. The parameters h_G, h_B solve the following non-linear system of equations (e.g. Bhamra, Kuehn, and Strebulaev 2010):

$$0 = \rho \Big(\frac{1-\omega}{1-\delta} \Big) h_i^{\delta-\omega} + \Big((1-\omega)\, \theta_i - \tfrac{1}{2} \omega (1-\omega) \left(\sigma_i^C \right)^2 - \rho \frac{1-\omega}{1-\delta} \Big) h_i^{1-\omega} + \lambda_i \left(h_j^{1-\omega} - h_i^{1-\omega} \right) \tag{A.5}$$

The parameters κ_i denote the relative jump sizes of the discount factor when the Markov chain leaves state i. The risk-free rates r_i contain the interest rate if the economy stayed in state i forever, $\bar{r}_i$, plus a second term that incorporates a possible switch in the state. The no-jump part of the interest rates, $\bar{r}_i$, are

$$\bar{r}_i = \rho + \delta \theta_i - \frac{1}{2} \omega (1 + \delta) \left(\sigma_i^C \right)^2, \tag{A.6}$$

and

$$w := e^{\kappa_B} = e^{-\kappa_G} \tag{A.7}$$

measures the size of the jump in the real-state price density when the economy shifts from bad states to good states (see for example Proposition 1 in Bhamra, Kuehn, and Strebulaev 2010).

Finally, r_i^p reflects the perpetual risk-free rate given by

$$r_i^p = r_i + \frac{r_j - r_i}{\tilde{p} + r_j} \tilde{p} \tilde{f}_j. \tag{A.8}$$

Appendix A.2. Derivation of the values of corporate securities after investment

The valuation of corporate debt. Our valuation of corporate debt of a firm that consists of only invested assets in a two state setting follows Hackbarth, Miao, and Morellec (2006) and Arnold, Wagner, and Westermann (2013). We illustrate the case in which the default boundary in good states is lower than that in bad states, i.e., $\hat{D}_G < \hat{D}_B$. If a firm defaults, debtholders receive a fraction $\Xi_i \alpha_i$ of the unlevered after tax asset value $(1-\tau)Xy_i$. A debt investor requires an instantaneous return equal to the risk-free rate r_i. The instantaneous debt return corresponds to the realized rate of return plus the coupon proceeds from debt. Hence, an application of Ito's lemma with possible state switches shows that debt satisfies the following system of ODEs.

For $0 \leq X \leq \hat{D}_G$:

$$\begin{cases} \hat{d}_G(X) &= \alpha_G \Xi_G (1-\tau) X y_G \\ \hat{d}_B(X) &= \alpha_B \Xi_B (1-\tau) X y_B. \end{cases} \tag{A.9}$$

For $\hat{D}_G < X \leq \hat{D}_B$:

$$\begin{cases} r_G \hat{d}_G(X) &= c + \tilde{\mu}_G X \hat{d}'_G(X) + \frac{1}{2}\tilde{\sigma}_G^2 X^2 \hat{d}''_G(X) + \tilde{\lambda}_G \left(\alpha_B \Xi_B (1-\tau) X y_B - \hat{d}_G(X) \right) \\ \hat{d}_B(X) &= \alpha_B \Xi_B (1-\tau) X y_B. \end{cases} \tag{A.10}$$

For $X > \hat{D}_B$:

$$\begin{cases} r_G \hat{d}_G(X) &= c + \tilde{\mu}_G X \hat{d}'_G(X) + \frac{1}{2}\tilde{\sigma}_G^2 X^2 \hat{d}''_G(X) + \tilde{\lambda}_G \left(\hat{d}_B(X) - \hat{d}_G(X) \right) \\ r_B \hat{d}_B(X) &= c + \tilde{\mu}_B X \hat{d}'_B(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 \hat{d}''_B(X) + \tilde{\lambda}_B \left(\hat{d}_G(X) - \hat{d}_B(X) \right). \end{cases} \tag{A.11}$$

The boundary conditions read

$$\lim_{X \to \infty} \frac{\hat{d}_i(X)}{X} < \infty, \quad i = G, B, \tag{A.12}$$

$$\lim_{X \searrow \hat{D}_B} \hat{d}_G(X) = \lim_{X \nearrow \hat{D}_B} \hat{d}_G(X), \tag{A.13}$$

$$\lim_{X \searrow \hat{D}_B} \hat{d}'_G(X) = \lim_{X \nearrow \hat{D}_B} \hat{d}'_G(X), \tag{A.14}$$

$$\lim_{X \searrow \hat{D}_G} \hat{d}_G(X) = \alpha_G \Xi_G (1-\tau) \hat{D}_G y_G, \tag{A.15}$$

and

$$\lim_{X \searrow \hat{D}_B} \hat{d}_B(X) = \alpha_B \Xi_B (1-\tau) \hat{D}_B y_B. \tag{A.16}$$

Condition (A.12) captures the no-bubbles condition. The other boundary conditions are the continuity condition (A.13), the smoothness condition (A.14), and the value-matching conditions (A.15) and (A.16) at the default thresholds $\hat{D}_G$ and $\hat{D}_B$ for the debt values in a good state and a bad state, $\hat{d}_G(\cdot)$ and $\hat{d}_B(\cdot)$. The functional form of the solution is

$$\hat{d}_i(X) = \begin{cases} \alpha_i \Xi_i (1-\tau) X y_i & X \leq \hat{D}_i & i = G, B \\ \hat{C}_1 X^{\beta_1^G} + \hat{C}_2 X^{\beta_2^G} + C_3 X + C_4 & \hat{D}_G < X \leq \hat{D}_B, & i = G \\ \hat{A}_{i1} X^{\gamma_1} + \hat{A}_{i2} X^{\gamma_2} + A_{i5} & X > \hat{D}_B, & i = G, B, \end{cases} \tag{A.17}$$

in which $\hat{A}_{G1}, \hat{A}_{G2}, \hat{A}_{B1}, \hat{A}_{B2}, A_{G5}, A_{B5}, \hat{C}_1, \hat{C}_2, C_3, C_4, \gamma_1, \gamma_2, \beta_1^G$, and β_2^G are real-valued parameters to be determined.

First, we consider the region $X > \hat{D}_B$. We start with the standard approach by plugging the functional form

$$\hat{d}_i(X) = \hat{A}_{i1} X^{\gamma_1} + \hat{A}_{i2} X^{\gamma_2} + A_{i5} \tag{A.18}$$

into both equations of (A.11). Comparing coefficients and solving the resulting two-dimensional system of equations for A_{i5}, we obtain

$$A_{i5} = \frac{c\,(r_j + \tilde{\lambda}_i + \tilde{\lambda}_j)}{r_i r_j + r_j \tilde{\lambda}_i + r_i \tilde{\lambda}_j} = \frac{c}{r_i^p}, \tag{A.19}$$

and find that $\hat{A}_{Gk}$ is a multiple of $\hat{A}_{Bk}$, $k = 1, 2$, with the factor $l_k := \frac{1}{\tilde{\lambda}_G}(r_G + \tilde{\lambda}_G - \tilde{\mu}_G \gamma_k - \frac{1}{2}\tilde{\sigma}_G^2 \gamma_k(\gamma_k - 1))$, i.e., $\hat{A}_{Bk} = l_k \hat{A}_{Gk}$. Using these results when comparing coefficients again, it can be shown that γ_1 and γ_2 are the negative roots of the quadratic equation

$$(\tilde{\mu}_B \gamma + \tfrac{1}{2}\tilde{\sigma}_B^2 \gamma(\gamma - 1) - \tilde{\lambda}_B - r_B)(\tilde{\mu}_G \gamma + \tfrac{1}{2}\tilde{\sigma}_G^2 \gamma(\gamma - 1) - \tilde{\lambda}_G - r_G) = \tilde{\lambda}_B \tilde{\lambda}_G. \tag{A.20}$$

To satisfy the no-bubbles condition for debt in Equation (A.12), we take the negative roots.

Next, we consider the region $\hat{D}_G \leq X \leq \hat{D}_B$. Plugging the functional form

$$d_G(X) = \hat{C}_1 X^{\beta_1^G} + \hat{C}_2 X^{\beta_2^G} + C_3 X + C_4 \tag{A.21}$$

into the first equation of (A.10), we find by comparison of coefficients that

$$\begin{aligned}
\beta_{1,2}^{G} &= \frac{1}{2} - \frac{\tilde{\mu}_G}{\tilde{\sigma}_G^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}_G}{\tilde{\sigma}_G^2}\right)^2 + \frac{2(r_G + \tilde{\lambda}_G)}{\tilde{\sigma}_G^2}} \\
C_3 &= \frac{\tilde{\lambda}_G \alpha_B \Xi_B (1-\tau) y_B}{r_G + \tilde{\lambda}_G - \tilde{\mu}_G} \\
C_4 &= \frac{c}{r_G + \tilde{\lambda}_G}.
\end{aligned} \tag{A.22}$$

We then plug the functional form (A.17) into conditions (A.13)–(A.16), and obtain a four-dimensional linear system in the remaining four unknown parameters $\hat{A}_{G1}$, $\hat{A}_{G2}$, $\hat{C}_1$, and $\hat{C}_2$:

$$\begin{aligned}
\hat{A}_{G1}\hat{D}_B^{\gamma_1} + \hat{A}_{G2}\hat{D}_B^{\gamma_2} + A_{G5} &= \hat{C}_1\hat{D}_G^{\beta_1^G} + \hat{C}_2\hat{D}_B^{\beta_2^G} + C_3\hat{D}_B + C_4 \\
\hat{A}_{G1}\gamma_1\hat{D}_B^{\gamma_1} + \hat{A}_{G2}\gamma_2\hat{D}_B^{\gamma_2} &= \hat{C}_1\beta_1^G\hat{D}_B^{\beta_1^G} + \hat{C}_2\beta_2^G\hat{D}_B^{\beta_2^G} + C_3\hat{D}_B \\
\alpha_G\Xi_G(1-\tau)\hat{D}_G y_G &= \hat{C}_1\hat{D}_B^{\beta_1^G} + \hat{C}_2\hat{D}_B^{\beta_2^G} + C_3\hat{D}_B + C_4 \\
l_1\hat{A}_{G1}\hat{D}_B^{\gamma_1} + l_2\hat{A}_{G2}\hat{D}_B^{\gamma_2} + A_{B5} &= \alpha_B\Xi_B(1-\tau)\hat{D}_B y_B.
\end{aligned} \tag{A.23}$$

We define the matrices

$$\hat{M} := \begin{bmatrix} \hat{D}_B^{\gamma_1} & \hat{D}_B^{\gamma_2} & -\hat{D}_B^{\beta_1^G} & -\hat{D}_B^{\beta_2^G} \\ \gamma_1\hat{D}_B^{\gamma_1} & \gamma_2\hat{D}_B^{\gamma_2} & -\beta_1^G\hat{D}_B^{\beta_1^G} & -\beta_2^G\hat{D}_B^{\beta_2^G} \\ 0 & 0 & \hat{D}_B^{\beta_1^G} & \hat{D}_B^{\beta_2^G} \\ l_1\hat{D}_B^{\gamma_1} & l_2\hat{D}_B^{\gamma_2} & 0 & 0 \end{bmatrix} \tag{A.24}$$

and

$$\hat{b} := \begin{bmatrix} C_3\hat{D}_B + C_4 - A_{G5} \\ C_3\hat{D}_B \\ \alpha_G\Xi_G(1-\tau)\hat{D}_G y_G - C_3\hat{D}_B - C_4 \\ \alpha_B\Xi_B(1-\tau)\hat{D}_B y_B - A_{B5} \end{bmatrix}, \tag{A.25}$$

such that $\hat{M}\begin{bmatrix}\hat{A}_{G1} & \hat{A}_{G2} & \hat{C}_1 & \hat{C}_2\end{bmatrix}^T = \hat{b}$. The solution for the unknown parameters is

$$\begin{bmatrix}\hat{A}_{G1} & \hat{A}_{G2} & \hat{C}_1 & \hat{C}_2\end{bmatrix}^T = \hat{M}^{-1}\hat{b}. \tag{A.26}$$

□

The value of the tax shield is calculated by using the formula for the value of debt, in which c is replaced by τc, and α is equal to zero. Similarly, we obtain the value of bankruptcy costs by simply replacing c with zero, and α with $1-\alpha$.

Default policy. The value of equity equals firm value minus the value of debt. Firm value is given by the value of assets in place plus the value of the growth option and the tax shield minus default costs. After debt has been issued, firms choose the ex post default policy that maximizes the value of equity. Formally, the default policy is determined by equating the first derivative of the equity value to zero at the corresponding default boundary level:

$$\begin{cases} \hat{e}'_G(\hat{D}^*_G) &= 0 \\ \hat{e}'_B(\hat{D}^*_B) &= 0. \end{cases} \tag{A.27}$$

We solve this problem numerically.

For a firm that receives scaled earnings after investment, the value of corporate securities is solved analogously by replacing X with the scaled earnings. For example, if the firm exercises the option in the good state and finances the exercise cost by issuing equity, the scaled earnings correspond to $(s_G + 1)X$. The default boundaries $\hat{D}^*_G$ and $\hat{D}^*_B$ are then expressed in terms of the scaled earnings levels.

Appendix A.3. Derivation of the value of the growth option

The case with $X_G < X_B$:

We present the derivation of the value of corporate securities for a firm that issues equity in good states and sells assets in bad states. For each state i, the option is exercised immediately whenever $X \geq X_i$ (option exercise region); otherwise, it is optimal to wait (option continuation region). This structure results in the following system of ODEs for the option's value function.

For $0 \leq X < X_G$:

$$\begin{cases} r_G G_G(X) &= \tilde{\mu}_G X G'_G(X) + \frac{1}{2}\tilde{\sigma}^2_G X^2 G''_G(X) + \tilde{\lambda}_G \left(G_B(X) - G_G(X)\right) \\ r_B G_B(X) &= \tilde{\mu}_B X G'_B(X) + \frac{1}{2}\tilde{\sigma}^2_B X^2 G''_B(X) + \tilde{\lambda}_B \left(G_G(X) - G_B(X)\right). \end{cases} \tag{A.28}$$

For $X_G \leq X < X_B$:

$$\begin{cases} G_G(X) &= (1-\tau)s_G X y_G - k_G(1+\phi_G) \\ r_B G_B(X) &= \tilde{\mu}_B X G'_B(X) + \frac{1}{2}\tilde{\sigma}^2_B X^2 G''_B(X) + \tilde{\lambda}_B \left((1-\tau)s_G X y_G - k_B(1+\phi_B) - G_B(X)\right). \end{cases} \tag{A.29}$$

For $X \geq X_B$:

$$\begin{cases} G_G(X) &= (1-\tau)s_G X y_G - k_G(1+\phi_G) \\ G_B(X) &= (1-\tau)s_B X y_B - k_B/\Xi_B. \end{cases} \tag{A.30}$$

When X is in the option continuation region, which corresponds to system (A.28) and the second

equation of (A.29), the required rate of return r_i (left-hand side) must be equal to the realized rate of return (right-hand side). We calculate the realized rate of return by using Ito's lemma for state switches. In this region, the last term captures the possible jump in the value of the growth option due to a state switch. It can be expressed as the instantaneous probability of a shift in the state, that is, $\tilde{\lambda}_G$ or $\tilde{\lambda}_B$, times the corresponding change in the value of the option. The first equation of (A.29) and the system (A.30) describe the payoff of the option at exercise. The process is in the option exercise region in these cases. The boundary conditions are

$$\lim_{X \searrow 0} G_i(X) = 0, \quad i = G, B, \tag{A.31}$$

$$\lim_{X \searrow X_G} G_B(X) = \lim_{X \nearrow X_G} G_B(X), \tag{A.32}$$

$$\lim_{X \searrow X_G} G_B'(X) = \lim_{X \nearrow X_G} G_B'(X), \tag{A.33}$$

$$\lim_{X \nearrow X_B} G_B(X) = (1-\tau) s_B X_B y_B - k_B / \Xi_B, \tag{A.34}$$

and

$$\lim_{X \nearrow X_G} G_G(X) = (1-\tau) s_G X_G y_G - k_G (1 + \phi_G). \tag{A.35}$$

Equation (A.31) ensures that the option value goes to zero as earnings approach zero. Equations (A.32) and (A.33) are the continuity and smoothness conditions of the value function in bad states at the exercise boundary in good states. Equations (A.34) and (A.35) are the value-matching conditions at the exercise boundaries in a good state and a bad state, respectively.

The functional form of the solution is

$$G_i(X) = \begin{cases} \bar{A}_{i3} X^{\gamma_3} + \bar{A}_{i4} X^{\gamma_4} & 0 \le X < X_G, \quad i = G, B \\ \bar{C}_1 X^{\beta_1^B} + \bar{C}_2 X^{\beta_2^B} + \bar{C}_3 X + \bar{C}_4 & X_G \le X < X_B, \quad i = B \\ (1-\tau) s_B X y_B - k_B / \Xi_B & X \ge X_B \quad i = B \\ (1-\tau) s_G X y_G - k_G (1 + \phi_G) & X \ge X_G \quad i = G, \end{cases} \tag{A.36}$$

in which $\bar{A}_{G3}, \bar{A}_{G4}, \bar{A}_{B1}, \bar{A}_{B2}, \bar{C}_1, \bar{C}_2, \bar{C}_3, \bar{C}_4, \gamma_3, \gamma_4, \beta_1^B$, and β_2^B are real-valued parameters that need to be determined.

First, we consider the region $0 \le X < X_G$ and plug the functional form $G_i(X) = \bar{A}_{i3} X^{\gamma_3} + \bar{A}_{i4} X^{\gamma_4}$ into both equations of (A.28). A comparison of coefficients implies that $\bar{A}_{Gk}$ is a multiple of $\bar{A}_{Bk}, k = 3, 4$, with the multiple factor $\bar{l}_k := \frac{1}{\tilde{\lambda}_G}(r_G + \tilde{\lambda}_G - \tilde{\mu}_G \gamma_k - \frac{1}{2}\tilde{\sigma}_G^2 \gamma_k(\gamma_k - 1))$, i.e., $\bar{A}_{Bk} = \bar{l}_k \bar{A}_{Gk}$.

Using this result when comparing coefficients, we find that γ_3 and γ_4 are the positive roots of the quadratic equation

$$(\tilde{\mu}_B\gamma + \frac{1}{2}\tilde{\sigma}_B^2\gamma(\gamma-1) - \tilde{\lambda}_B - r_B)(\tilde{\mu}_G\gamma + \frac{1}{2}\tilde{\sigma}_G^2\gamma(\gamma-1) - \tilde{\lambda}_G - r_G) = \tilde{\lambda}_B\tilde{\lambda}_G. \tag{A.37}$$

Boundary condition (A.31) implies to use the positive roots.

Next, we consider the region $X_G \leq X < X_B$. Plugging the functional form $G_B(X) = \bar{C}_1 X^{\beta_1} + \bar{C}_2 X^{\beta_2} + \bar{C}_3 X + \bar{C}_4$ into the second equation of (A.29), we find by comparison of coefficients that

$$\begin{aligned}
\beta_{1,2}^B &= \frac{1}{2} - \frac{\tilde{\mu}_B}{\tilde{\sigma}_B^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}_B}{\tilde{\sigma}_B^2}\right)^2 + \frac{2(r_B + \tilde{\lambda}_B)}{\tilde{\sigma}_B^2}}, \\
\bar{C}_3 &= \tilde{\lambda}_B \frac{(1-\tau)s_G y_G}{r_B - \tilde{\mu}_B + \tilde{\lambda}_B}, \\
\bar{C}_4 &= -\tilde{\lambda}_B \frac{k_B/\Xi_B}{r_B + \tilde{\lambda}_B}.
\end{aligned} \tag{A.38}$$

The remaining unknown parameters are $\bar{A}_{G3}, \bar{A}_{G4}, \bar{C}_1$, and $\bar{C}_2$. Plugging the functional form (A.36) into conditions (A.32)–(A.35) yields

$$\bar{C}_1 X_G^{\beta_1^B} + \bar{C}_2 X_G^{\beta_2^B} + \bar{C}_3 X_G + \bar{C}_4 = \bar{l}_3 \bar{A}_{G3} X_G^{\gamma_3} + \bar{l}_4 \bar{A}_{G4} X_G^{\gamma_4}, \tag{A.39}$$

$$\bar{C}_1 \beta_1^B X_G^{\beta_1^B} + \bar{C}_2 \beta_2^B X_G^{\beta_2^B} + \bar{C}_3 X_G = \bar{l}_3 \bar{A}_{G3} \gamma_3 X_G^{\gamma_3} + \bar{l}_4 \gamma_4 \bar{A}_{G4} X_G^{\gamma_4}, \tag{A.40}$$

$$\bar{C}_1 X_B^{\beta_1^B} + \bar{C}_2 X_B^{\beta_2^B} + \bar{C}_3 X_B + \bar{C}_4 = (1-\tau)s_B y_B X_B - k_B/\Xi_B, \tag{A.41}$$

and

$$\bar{A}_{G3} X_G^{\gamma_3} + \bar{A}_{G4} X_G^{\gamma_4} = (1-\tau)s_G y_G X_G - k_G(1+\phi_G). \tag{A.42}$$

This four-dimensional system is linear in its four unknowns $\bar{A}_{G3}, \bar{A}_{G4}, \bar{C}_1$ and $\bar{C}_2$. We define the matrices

$$\bar{M} := \begin{bmatrix} \bar{l}_3 X_G^{\gamma_3} & \bar{l}_4 X_G^{\gamma_4} & -X_G^{\beta_1^B} & -X_G^{\beta_2^B} \\ \bar{l}_3 \gamma_3 X_G^{\gamma_3} & \bar{l}_4 \gamma_4 X_G^{\gamma_4} & -\beta_1^B X_G^{\beta_1^B} & -\beta_2^B X_G^{\beta_2^B} \\ 0 & 0 & X_B^{\beta_1^B} & X_B^{\beta_2^B} \\ X_G^{\gamma_3} & X_G^{\gamma_4} & 0 & 0 \end{bmatrix}, \tag{A.43}$$

and

$$\bar{b} := \begin{bmatrix} \bar{C}_3 X_G + \bar{C}_4 \\ \bar{C}_3 X_G \\ -\bar{C}_3 X_B - \bar{C}_4 + (1-\tau) s_B y_B X_B - k_B/\Xi_B \\ (1-\tau) s_G y_G X_G - k_G(1+\phi_G) \end{bmatrix}, \tag{A.44}$$

such that $\bar{M}\begin{bmatrix}\bar{A}_{G3} & \bar{A}_{G4} & \bar{C}_1 & \bar{C}_2\end{bmatrix}^T = \bar{b}$. The solution to the remaining four unknowns is

$$\begin{bmatrix}\bar{A}_{G3} & \bar{A}_{G4} & \bar{C}_1 & \bar{C}_2\end{bmatrix}^T = \bar{M}^{-1}\bar{b}. \tag{A.45}$$

□

The unlevered value of the growth option. The unlevered value of the growth option corresponds to the value of an option in a firm without debt (all equity firm). It is calculated by additionally imposing the smooth-pasting boundary conditions at option exercise:

$$\lim_{X \nearrow X_B^{unlev}} G_B^{unlev\prime}(X) = (1-\tau) s_B y_B \tag{A.46}$$

and

$$\lim_{X \nearrow X_G^{unlev}} G_G^{unlev\prime}(X) = (1-\tau) s_G y_G. \tag{A.47}$$

The solution method is analog to that for the levered growth option value up to and including Equation (A.38). The system (A.39)–(A.42) needs to be augmented by the two equations corresponding to the additional smooth-pasting boundary conditions:

$$\bar{C}_1^{unlev} \beta_1^B \left(X_B^{unlev}\right)^{\beta_1^B - 1} + \bar{C}_2^{unlev} \beta_2^B \left(X_B^{unlev}\right)^{\beta_2^B - 1} + \bar{C}_3 = (1-\tau) s_B y_B \tag{A.48}$$

and

$$\bar{A}_{G3}^{unlev} \gamma_3 \left(X_G^{unlev}\right)^{\gamma_3 - 1} + \bar{A}_{G4}^{unlev} \gamma_4 \left(X_G^{unlev}\right)^{\gamma_4 - 1} = (1-\tau) s_G y_G. \tag{A.49}$$

The full system is six-dimensional with the six unknowns $\bar{A}_{G3}^{unlev}, \bar{A}_{G4}^{unlev}, \bar{C}_1^{unlev}, \bar{C}_2^{unlev}$ X_G^{unlev}, and X_B^{unlev}, linear in the first four unknowns and nonlinear in the last two unknowns. We solve this system numerically.

The case with $X_G \geq X_B$:

The solution of the case $X_G \geq X_B$ is obtained immediately by renaming states in the solution of the presented case for $X_G < X_B$.

Alternative financing strategies:

Values of growth options for any alternative financing strategy can be derived analogously by replacing the friction adjusted option exercise cost terms in Equations (A.29), (A.30), (A.34), and (A.35).

Appendix A.4. Firms with invested assets and an expansion option

We first present a proof for the value of corporate debt in the case in which $D_G < D_B$, $\hat{D}_G < \hat{D}_B$, and $X_B > X_G$. The argument of the proof is adapted from Arnold, Wagner, and Westermann (2013).

Proof of Proposition 2. A debt investor requires an instantaneous return equal to the risk-free rate r_i. The application of Ito's lemma with state switches shows that debt must satisfy the following system of ODEs.

For $0 \leq X \leq D_G$:

$$\begin{cases} d_G(X) &= \alpha_G \Xi_G \left((1-\tau) X y_G + G_G^{unlev}(X)\right) \\ d_B(X) &= \alpha_B \Xi_B \left((1-\tau) X y_B + G_B^{unlev}(X)\right). \end{cases} \tag{A.50}$$

For $D_G < X \leq D_B$:

$$\begin{cases} r_G d_G(X) &= c + \tilde{\mu}_G X d_G'(X) + \frac{1}{2}\tilde{\sigma}_G^2 X^2 d_G''(X) \\ & \quad + \tilde{\lambda}_G \left(\alpha_B \phi_B \left((1-\tau) X y_B + G_B^{unlev}(X)\right) - d_G(X)\right) \\ d_B(X) &= \alpha_B \Xi_B \left((1-\tau) X y_B + G_B^{unlev}(X)\right). \end{cases} \tag{A.51}$$

For $D_B < X < X_G$:

$$\begin{cases} r_G d_G(X) &= c + \tilde{\mu}_G X d_G'(X) + \frac{1}{2}\tilde{\sigma}_G^2 X^2 d_G''(X) + \tilde{\lambda}_G \left(d_B(X) - d_G(X)\right) \\ r_B d_B(X) &= c + \tilde{\mu}_B X d_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 d_B''(X) + \tilde{\lambda}_B \left(d_G(X) - d_B(X)\right). \end{cases} \tag{A.52}$$

For $X_G \leq X < X_B$:

$$\begin{cases} d_G(X) &= \hat{d}_G\left((s_G+1)X\right) \\ r_B d_B(X) &= c + \tilde{\mu}_B X d_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 d_B''(X) + \tilde{\lambda}_B \left(\hat{d}_G\left((s_G+1)X\right) - d_B(X)\right). \end{cases} \tag{A.53}$$

For $X \geq X_B$:

$$\begin{cases} d_G(X) &= \hat{d}_G((s_G+1)X) \\ d_B(X) &= \hat{d}_B\left((s_B+1-\dfrac{k_B/\Xi_B}{(1-\tau)X_{\bar{i}}y_B})X\right). \end{cases} \tag{A.54}$$

In system (A.50), the firm is in the default region in both good and bad states. In this region, debtholders receive at default

$$\alpha_i \Xi_i \left((1-\tau)Xy_i + G_i^{unlev}(X)\right).$$

The firm is in the continuation region in good states and in the default region in bad states in system (A.51). For the continuation region in good states, the left-hand side of the first equation is the instantaneous rate of return required by investors for holding corporate debt. The right-hand side is the realized rate of return, computed by Ito's lemma as the expected change in the value of debt plus the coupon payment c. The last term expresses the possible jump in the value of debt in case of a state switch that triggers immediate default. Equations (A.52) describe the case in which the firm is in the continuation region in both good and bad states. The next system, (A.53), treats the case in which the firm is in the exercise region in good states and in the continuation region in bad states. After exercising the option, the firm owns total assets in place with value

$$(1-\tau)Xy_i + (1-\tau)s_{\bar{i}}Xy_i, \tag{A.55}$$

reflecting the notion that the exercise cost of the growth option can be financed by issuing equity in good states. The value of debt must then be equal to the value of debt of a firm with only invested assets, i.e.,

$$d_G(X) = \hat{d}_G((s_G+1)\,X), \tag{A.56}$$

which is the first equation in (A.53). We obtain the second equation in this case by using the same approach as in (A.52). The last term captures the notion that a switch from bad states to good states triggers immediate exercise of the expansion option with equity financing. Finally, (A.54) describes the case in which the firm is in the exercise region in both good and bad states. In good states, the earnings of the firm are scaled by $s_G + 1$. In bad states, the exercise cost k_B is financed by selling

$$\frac{k_B/\Xi_B}{(1-\tau)X_{\bar{i}}y_B} \tag{A.57}$$

of the assets in place, such that the earnings of the firm are scaled by

$$(s_B + 1 - \frac{k_B/\Xi_B}{(1-\tau)X_{\bar{t}}y_B}). \tag{A.58}$$

The system is subject to the following boundary conditions.

$$\lim_{X \searrow D_B} d_G(X) = \lim_{X \nearrow D_B} d_G(X), \tag{A.59}$$

$$\lim_{X \searrow D_B} d'_G(X) = \lim_{X \nearrow D_B} d'_G(X), \tag{A.60}$$

$$\lim_{X \searrow D_G} d_G(X) = \alpha_G \Xi_G \left((1-\tau)D_G y_G + G_G^{unlev}(D_G)\right), \tag{A.61}$$

$$\lim_{X \searrow D_B} d_B(X) = \alpha_B \Xi_B \left((1-\tau)D_B y_B + G_B^{unlev}(D_B)\right), \tag{A.62}$$

$$\lim_{X \searrow X_G} d_B(X) = \lim_{X \nearrow X_G} d_B(X), \tag{A.63}$$

$$\lim_{X \searrow X_G} d'_B(X) = \lim_{X \nearrow X_G} d'_B(X), \tag{A.64}$$

$$\lim_{X \nearrow X_G} d_G(X) = \hat{d}_G((s_G+1)X_G), \tag{A.65}$$

and

$$\lim_{X \nearrow X_B} d_B(X_B) = \hat{d}_B\left((s_B + 1 - \frac{k_B/\Xi_B}{(1-\tau)X_{\bar{t}}y_B})X_B\right). \tag{A.66}$$

Equations (A.59) and (A.60) are the continuity and smoothness conditions for the debt value in the good state at the default boundary of the bad state. Equations (A.61) and (A.62) show the value-matching conditions at the default thresholds, and Equations (A.63) and (A.64) reflect the continuity and smoothness conditions for the debt value in the bad state at the option exercise boundary of the good state. Equations (A.65) and (A.66) are the value-matching conditions at the option exercise boundaries in the good state and the bad state. The default thresholds and option exercise boundaries are chosen by equityholders. Hence, we do not need the corresponding smooth-pasting conditions for debt.

To solve this system, we start with the functional form of the solution, in which $A_{G1}, A_{G2}, A_{B1}, A_{B2}, C_1, C_2, C_3, C_4, C_5, C_6, B_1, B_2, B_4, \beta_1^G, \beta_2^G, \beta_1^B, \beta_2^B, \gamma_1, \gamma_2, \gamma_3$, and γ_4 are real-valued parameters to be determined.

We first consider the region $D_B < X \leq X_G$. Plugging the functional form $d_i(X) = A_{i1}X^{\gamma_1} + A_{i2}X^{\gamma_2} + A_{i3}X^{\gamma_3} + A_{i4}X^{\gamma_4} + A_{i5}$ into both equations of (A.52) and comparing coefficients, we ob-

tain

$$A_{i5} = \frac{c(r_j + \tilde{\lambda}_i + \tilde{\lambda}_j)}{r_i r_j + r_j \tilde{\lambda}_i + r_i \tilde{\lambda}_j} = \frac{c}{r_i^p}. \tag{A.67}$$

As in Appendix A.2, A_{Gk} is a multiple of A_{Bk}, $k = 1, \ldots, 4$, with the multiple factor $l_k := \frac{1}{\tilde{\lambda}_G}(r_G + \tilde{\lambda}_G - \tilde{\mu}_G \gamma_k - \frac{1}{2}\tilde{\sigma}_G^2 \gamma_k(\gamma_k - 1))$, i.e., $A_{Bk} = l_k A_{Gk}$. Using this relation and comparing coefficients, it can be shown that $\gamma_1, \gamma_2, \gamma_3$, and γ_4 correspond to the roots of the quadratic equation

$$(\tilde{\mu}_B \gamma + \tfrac{1}{2}\tilde{\sigma}_B^2 \gamma(\gamma - 1) - \tilde{\lambda}_B - r_B)(\tilde{\mu}_G \gamma + \tfrac{1}{2}\tilde{\sigma}_G^2 \gamma(\gamma - 1) - \tilde{\lambda}_G - r_G) = \tilde{\lambda}_B \tilde{\lambda}_G. \tag{A.68}$$

According to Guo (2001), this quadratic equation always has two negative and two positive distinct real roots. The value of debt in both states is subject to boundary conditions from below (default) and above (exercise of expansion option). To satisfy these boundary conditions, we use four terms with the corresponding factors A_{ik} as well as the exponents γ_k, which requires the usage of all four roots of Equation (A.68). We do not incorporate the no-bubbles condition again because it is already implemented in the value function $\hat{d}_i$ of a firm with only invested assets. The unknown parameters for this region are A_{Gk}, $k = 1, \ldots, 4$.

Next, we examine the region $D_G \leq X \leq D_B$. Plugging the functional form $d_G(X) = C_1 X^{\beta_1^G} + C_2 X^{\beta_2^G} + C_3 X + C_4 + C_5 X^{\gamma_3} + C_6 X^{\gamma_4}$ into the second equation of (A.51), we find by comparison of coefficients that

$$\beta_{1,2}^G = \frac{1}{2} - \frac{\tilde{\mu}_G}{\tilde{\sigma}_G^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}_G}{\tilde{\sigma}_G^2}\right)^2 + \frac{2(r_G + \tilde{\lambda}_G)}{\tilde{\sigma}_G^2}}, \tag{A.69}$$

$$C_3 = \tilde{\lambda}_G \frac{\alpha_B \Xi_B (1 - \tau) y_B}{r_G + \tilde{\lambda}_G - \tilde{\mu}_G}, \tag{A.70}$$

$$C_4 = \frac{c}{r_G + \tilde{\lambda}_G}, \tag{A.71}$$

$$C_5 = \alpha_B \Xi_B \frac{\bar{l}_3}{l_3} \bar{A}_{G3}^{unlev}, \tag{A.72}$$

and

$$C_6 = \alpha_B \Xi_B \frac{\bar{l}_4}{l_4} \bar{A}_{G4}^{unlev}. \tag{A.73}$$

The unknown parameters left in this region are C_1 and C_2.

Finally, we consider the region $X_G < X \leq X_B$. Plugging the functional form

$$B_1 X^{\beta_1^B} + B_2 X^{\beta_2^B} + Z(X) + \tilde{\lambda}_B \frac{c}{r_i^P \left(r_B + \tilde{\lambda}_B\right)} + \frac{c}{r_B + \tilde{\lambda}_B} \tag{A.74}$$

into the second equation of (A.53) and comparing coefficients, we find that

$$Z(X) = \tilde{\lambda}_B B_5 X^{\gamma_1} + \tilde{\lambda}_B B_6 X^{\gamma_2}. \tag{A.75}$$

The parameters B_5 and B_6 are given by

$$B_5 = \frac{(s_B + 1)^{\gamma_1} \hat{A}_{G1}}{r_B - \tilde{\mu}_B \gamma_1 - \frac{1}{2}\tilde{\sigma}_B^2 \gamma_1 (\gamma_1 - 1) + \tilde{\lambda}_B}, \tag{A.76}$$

and

$$B_6 = \frac{(s_B + 1)^{\gamma_2} \hat{A}_{G2}}{r_B - \tilde{\mu}_B \gamma_2 - \frac{1}{2}\tilde{\sigma}_B^2 \gamma_2 (\gamma_2 - 1) + \tilde{\lambda}_B}. \tag{A.77}$$

The unknown parameters in this region are B_1 and B_2.

To obtain the unknown parameters $A_{G1}, A_{G2}, A_{G3}, A_{G4}, C_1, C_2, B_1$, and B_2, we plug the functional form into the system of boundary conditions (A.59)–(A.66):

$$\begin{aligned}
\sum_{k=1}^{4} A_{Gk} D_B^{\gamma_k} + A_{G5} &= C_1 D_B^{\beta_1^G} + C_2 D_B^{\beta_2^G} + C_3 X + C_4 + C_5 X^{\gamma_3} + C_6 X^{\gamma_4} \\
\sum_{k=1}^{4} A_{Gk} \gamma_k D_B^{\gamma_k} &= C_1 \beta_1^G D_B^{\beta_1^G} + C_2 \beta_2^G D_B^{\beta_2^G} + C_3 X + C_5 \gamma_3 X^{\gamma_3} + C_6 \gamma_4 X^{\gamma_4} \\
\alpha_G \Xi_G \left((1+\tau) D_G y_G + G_G^{unlev}(D_G)\right) &= C_1 D_G^{\beta_1^G} + C_2 D_G^{\beta_2^G} + C_3 D_G + C_4 + C_5 D_G^{\gamma_3} + C_6 D_G^{\gamma_4} \\
\sum_{k=1}^{4} l_k A_{Gk} D_B^{\gamma_k} + A_{B5} &= \alpha_B \Xi_B \left((1+\tau) D_B y_B + G_B^{unlev}(D_B)\right) \\
\sum_{k=1}^{4} l_k A_{Gk} X_G^{\gamma_k} + A_{B5} &= B_1 X_G^{\beta_1^B} + B_2 X_G^{\beta_2^B} + Z(X_G) + B_4 \\
\sum_{k=1}^{4} l_k A_{Gk} \gamma_k X_G^{\gamma_k} &= B_1 \beta_1^B X_G^{\beta_1^B} + B_2 \beta_2^B X_G^{\beta_2^B} + X_G Z'(X_G) \\
\sum_{k=1}^{4} A_{Gk} X_G^{\gamma_k} + A_{G5} &= \hat{d}_G \left((s_G + 1) X_G\right) \\
B_1 X_B^{\beta_1^B} + B_2 X_B^{\beta_2^B} + Z(X_B) + B_4 &= \hat{d}_B \left((s_B + 1 - \frac{k_B / \Xi_B}{(1-\tau) X_{\bar{t}} y_B}) X_B\right).
\end{aligned} \tag{A.78}$$

Using matrix notation, we can write

$$M := \begin{bmatrix} D_B^{\gamma_1} & D_B^{\gamma_2} & D_B^{\gamma_3} & D_B^{\gamma_4} & -D_B^{\beta_1^G} & -D_B^{\beta_2^G} & 0 & 0 \\ \gamma_1 D_B^{\gamma_1} & \gamma_2 D_B^{\gamma_2} & \gamma_3 D_B^{\gamma_3} & \gamma_4 D_B^{\gamma_4} & -\beta_1^G D_B^{\beta_1^G} & -\beta_2^G D_B^{\beta_2^G} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_G^{\beta_1^G} & D_G^{\beta_2^G} & 0 & 0 \\ l_1 D_B^{\gamma_1} & l_2 D_B^{\gamma_2} & l_3 D_B^{\gamma_3} & l_4 D_B^{\gamma_4} & 0 & 0 & 0 & 0 \\ l_1 X_G^{\gamma_1} & l_2 X_G^{\gamma_2} & l_3 X_G^{\gamma_3} & l_4 X_G^{\gamma_4} & 0 & 0 & -X_G^{\beta_1^B} & -X_G^{\beta_2^B} \\ l_1 \gamma_1 X_G^{\gamma_1} & l_2 \gamma_2 X_G^{\gamma_2} & l_3 \gamma_3 X_G^{\gamma_3} & l_4 \gamma_4 X_G^{\gamma_4} & 0 & 0 & -\beta_1^B X_G^{\beta_1^B} & -\beta_2^B X_G^{\beta_2^B} \\ X_G^{\gamma_1} & X_G^{\gamma_2} & X_G^{\gamma_3} & X_G^{\gamma_4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & X_B^{\beta_1^B} & X_B^{\beta_2^B} \end{bmatrix} \tag{A.79}$$

and

$$b := \begin{bmatrix} -A_{G5} + C_3 D_B + C_4 + C_5 D_B^{\gamma_1} + C_6 D_B^{\gamma_2} \\ C_3 D_B + \gamma_1 C_5 D_B^{\gamma_1} + \gamma_2 C_6 D_B^{\gamma_2} \\ -C_3 D_G - C_4 - C_5 D_G^{\gamma_3} - C_6 D_G^{\gamma_4} + \alpha_G \Xi_G \left((1-\tau) D_G y_G + G_G^{unlev}(D_G) \right) \\ -A_{B5} + \alpha_B \Xi_B \left((1-\tau) D_B y_B + G_B^{unlev}(D_B) \right) \\ -A_{B5} + Z\left(X_G\right) + B_4 \\ X_G Z'\left(X_G\right) \\ -A_{G5} + \hat{d}_G \left((s_G + 1) X_G \right) \\ -Z\left(X_B\right) + B_4 + \hat{d}_B \left((s_B + 1 - \dfrac{k_B / \Xi_B}{(1-\tau) X_{\bar{t}} y_B}) X_B \right) \end{bmatrix}. \tag{A.80}$$

The solution to the remaining unknowns is

$$\begin{bmatrix} A_{G1} & A_{G2} & A_{G3} & A_{G4} & C_1 & C_2 & B_1 & B_2 \end{bmatrix}^T = M^{-1} b. \tag{A.81}$$

□

The case with $D_G < D_B$, $\hat{D}_G < \hat{D}_B$, and $X_G > X_B$:

Going through the same steps as in the previous case gives us

$$M := \begin{bmatrix} D_B^{\gamma_1} & D_B^{\gamma_2} & D_B^{\gamma_3} & D_B^{\gamma_4} & -D_B^{\beta_1^G} & -D_B^{\beta_2^G} & 0 & 0 \\ \gamma_1 D_B^{\gamma_1} & \gamma_2 D_B^{\gamma_2} & \gamma_3 D_B^{\gamma_3} & \gamma_4 D_B^{\gamma_4} & -\beta_1^G D_B^{\beta_1^G} & -\beta_2^G D_B^{\beta_2^G} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_G^{\beta_1^G} & D_G^{\beta_2^G} & 0 & 0 \\ l_1 D_B^{\gamma_1} & l_2 D_G^{\gamma_2} & l_3 D_B^{\gamma_3} & l_4 D_B^{\gamma_4} & 0 & 0 & 0 & 0 \\ X_B^{\gamma_1} & X_B^{\gamma_2} & X_B^{\gamma_3} & X_B^{\gamma_4} & 0 & 0 & -X_B^{\beta_1^G} & -X_B^{\beta_2^G} \\ \gamma_1 X_B^{\gamma_1} & \gamma_2 X_B^{\gamma_2} & \gamma_3 X_B^{\gamma_3} & \gamma_4 X_B^{\gamma_4} & 0 & 0 & -\beta_1^G X_B^{\beta_1^G} & -\beta_2^G X_B^{\beta_2^G} \\ l1 X_B^{\gamma_1} & l2 X_B^{\gamma_2} & l3 X_B^{\gamma_3} & l4 X_B^{\gamma_4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & X_G^{\beta_1^G} & X_G^{\beta_2^G} \end{bmatrix} \tag{A.82}$$

and

$$b := \begin{bmatrix} -A_{G5} + C_3 D_B + C_4 + C_5 D_B^{\gamma_1} + C_6 D_B^{\gamma_2} \\ C_3 D_B + \gamma_1 C_5 D_B^{\gamma_1} + \gamma_2 C_6 D_B^{\gamma_2} \\ -C_3 D_G - C_4 - C_5 D_G^{\gamma_3} - C_6 D_G^{\gamma_4} + \alpha_G \Xi_G \left((1-\tau) D_G y_G + G_G^{unlev}(D_G) \right) \\ -A_{B5} + \alpha_B \Xi_B \left((1-\tau) D_B y_B + G_B^{unlev}(D_B) \right) \\ -A_{G5} + Z\left(X_B\right) + B_4 \\ X_B Z'\left(X_B\right) \\ -A_{B5} + \hat{d}_B \left((s_B + 1 - \frac{k_B/\Xi_B}{(1-\tau) X_{\bar{t}} y_B}) X_B \right) \\ -Z\left(X_G\right) + B_4 + \hat{d}_G \left((s_G + 1) X_G \right) \end{bmatrix}. \tag{A.83}$$

The solution to the unknowns is again determined by

$$\begin{bmatrix} A_{G1} & A_{G2} & A_{G3} & A_{G4} & C_1 & C_2 & B_1 & B_2 \end{bmatrix}^T = M^{-1} b. \tag{A.84}$$

The case with $X_G \geq X_B$:

The solution of the case $X_G \geq X_B$ is obtained immediately by renaming states in the solution of the presented case for $X_G < X_B$.

Alternative financing strategies:

Debt values for any alternative financing strategy can be derived analogously by replacing earnings levels at option exercise in Equations (A.53), (A.54), (A.65), and (A.66).

□

Appendix A.5. Derivation of the value of bankruptcy costs

For the calculation of bankruptcy costs, the ODEs are given by the following system:

For $0 \leq X \leq D_G$:

$$\begin{cases} b_G(X) &= (1-\alpha_G\Xi_G)(1-\tau)Xy_G + G_G(X) - \alpha_G\Xi_G G_G^{unlev}(X) \\ b_B(X) &= (1-\alpha_B\Xi_B)(1-\tau)Xy_B + G_B(X) - \alpha_B\Xi_B B_B^{unlev}(X). \end{cases} \tag{A.85}$$

For $D_G < X \leq D_B$:

$$\begin{cases} r_G b_G(X) &= \tilde{\mu}_G X b_G'(X) + \frac{1}{2}\tilde{\sigma}_G^2 X^2 b_G''(X) \\ & \quad +\tilde{\lambda}_G\left((1-\alpha_B\Xi_B)(1-\tau)Xy_B + G_B(X) - \alpha_B\Xi_B G_B^{unlev}(X) - b_G(X)\right) \\ b_B(X) &= (1-\alpha_B\Xi_B)(1-\tau)Xy_B + G_B(X) - \alpha_B\Xi_B G_B^{unlev}(X). \end{cases} \tag{A.86}$$

For $D_B < X < X_G$:

$$\begin{cases} r_G d_G(X) &= c + \tilde{\mu}_G X b_G'(X) + \frac{1}{2}\tilde{\sigma}_G^2 X^2 b_G''(X) + \tilde{\lambda}_G(b_B(X) - b_G(X)) \\ r_B d_B(X) &= c + \tilde{\mu}_B X d_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 b_B''(X) + \tilde{\lambda}_B(b_G(X) - b_B(X)). \end{cases} \tag{A.87}$$

For $X_G \leq X < X_B$:

$$\begin{cases} b_G(X) &= \hat{d}_G((s_G+1)X) \\ rd_B(X) &= c + \tilde{\mu}_B X b_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 b_B''(X) + \tilde{\lambda}_B\left(\hat{d}_G((s_G+1)X) - b_B(X)\right). \end{cases} \tag{A.88}$$

For $X \geq X_B$:

$$\begin{cases} b_G(X) &= \hat{b}_G((s_G+1)X) \\ b_B(X) &= \hat{b}_B\left((s_B + 1 - \frac{k_B/\Xi_B}{(1-\tau)X_{\bar{t}}y_B})X\right). \end{cases} \tag{A.89}$$

The boundary conditions are as follows:

$$\lim_{X\searrow D_B} b_G(X) = \lim_{X\nearrow D_B} b_G(X), \tag{A.90}$$

$$\lim_{X\searrow D_B} b_G'(X) = \lim_{X\nearrow D_B} b_G'(X), \tag{A.91}$$

$$\lim_{X\searrow D_G} b_G(X) = (1-\alpha_G\Xi_B)(1-\tau)D_G y_G + G_G(D_G) - \alpha_G\Xi_G G_G^{unlev}(D_G), \tag{A.92}$$

$$\lim_{X\searrow D_B} b_B(X) = (1-\alpha_G\Xi_G)(1-\tau)D_B y_B + G_B(D_B) - \alpha_B G_B^{unlev}(D_B), \tag{A.93}$$

$$\lim_{X\searrow X_G} b_B(X) = \lim_{X\nearrow X_G} b_B(X), \tag{A.94}$$

$$\lim_{X\searrow X_G} b_B'(X) = \lim_{X\nearrow X_G} b_B'(X), \tag{A.95}$$

$$\lim_{X\nearrow X_G} b_G(X) = \hat{b}_G((s_G+1)X_G), \tag{A.96}$$

and

$$\lim_{X \nearrow X_B} b_B\left(X_B\right) = \hat{b}_B\left((s_B + 1 - \frac{k_B/\Xi_B}{(1-\tau)X_{\bar{i}}y_B})X_B\right). \tag{A.97}$$

Equations (A.90) and (A.91) are the continuity and smoothness conditions for bankruptcy costs in good states at the default boundary in bad states. Similarly, Equations (A.94) and (A.95) are the corresponding conditions for bankruptcy costs in bad states at the option exercise boundary in good states. Equations (A.92) and (A.93) are the value-matching conditions at the default thresholds. They incorporate the fact that upon default, the value of the levered growth option switches to the value of the unlevered growth option. Equations (A.96) and (A.97) are the value-matching conditions at the option exercise boundaries.

To solve for the unknown parameters, we plug the functional form

$$b_i\left(X\right) = \begin{cases} (1-\alpha_i\Xi_i)(1-\tau)Xy_i - \alpha_i\Xi_i G_i^{unlev}\left(X\right) + G_i(X) & X \le D_i, & i = G, B \\ C_1 X^{\beta_1^B} + C_2 X^{\beta_2^B} + C_5 X^{\gamma_3} + C_6 X^{\gamma_4} & & \\ \quad + \tilde{\lambda}_G \dfrac{\alpha_B \Xi_B y_B (1-\tau)}{r_G - \tilde{\mu}_G + \tilde{\lambda}_G} X + \dfrac{c}{r_G + \tilde{\lambda}_G} & D_G < X \le D_B, & i = G \\ A_{i1}X^{\gamma_1} + A_{i2}X^{\gamma_2} + A_{i3}X^{\gamma_3} + A_{i4}X^{\gamma_4} + \frac{c}{r_i^P} & D_B < X \le X_G, & i = G, B \\ B_1 X^{\beta_1^B} + B_2 X^{\beta_2^B} + Z\left(X\right) + \tilde{\lambda}_B \dfrac{c}{r_i^P(r_B + \tilde{\lambda}_B)} + \dfrac{c}{r_B + \tilde{\lambda}_B} & X_G < X \le X_B, & i = B \\ \hat{b}_G\left((s_G + 1)X\right) & X > X_G, & i = G \\ \hat{b}_B\left((s_B + 1 - \dfrac{k_B/\Xi_B}{(1-\tau)X_{\bar{i}}y_B})X\right) & X > X_B, & i = B \end{cases} \tag{A.98}$$

into the system of boundary conditions (A.90)–(A.97). The solution to the unknown parameters is given by

$$\begin{bmatrix} A_{G1} & A_{G2} & A_{G3} & A_{G4} & C_1 & C_2 & B_1 & B_2 \end{bmatrix}^T = M^{-1}b, \tag{A.99}$$

in which

$$M := \begin{bmatrix} D_B^{\gamma_1} & D_B^{\gamma_2} & D_B^{\gamma_3} & D_B^{\gamma_4} & -D_B^{\beta_1^G} & -D_B^{\beta_2^G} & 0 & 0 \\ \gamma_1 D_B^{\gamma_1} & \gamma_2 D_B^{\gamma_2} & \gamma_3 D_B^{\gamma_3} & \gamma_4 D_B^{\gamma_4} & -\beta_1^G D_B^{\beta_1^G} & -\beta_2^G D_B^{\beta_2^G} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_G^{\beta_1^G} & D_G^{\beta_2^G} & 0 & 0 \\ l_1 D_B^{\gamma_1} & l_2 D_B^{\gamma_2} & l_3 D_B^{\gamma_3} & l_4 D_B^{\gamma_4} & 0 & 0 & 0 & 0 \\ l_1 X_G^{\gamma_1} & l_2 X_G^{\gamma_2} & l_3 X_G^{\gamma_3} & l_4 X_G^{\gamma_4} & 0 & 0 & -X_G^{\beta_1^B} & -X_G^{\beta_2^B} \\ l_1 \gamma_1 X_G^{\gamma_1} & l_2 \gamma_2 X_G^{\gamma_2} & l_3 \gamma_3 X_G^{\gamma_3} & l_4 \gamma_4 X_G^{\gamma_4} & 0 & 0 & -\beta_1^B X_G^{\beta_1^B} & -\beta_2^B X_G^{\beta_2^B} \\ X_G^{\gamma_1} & X_G^{\gamma_2} & X_G^{\gamma_3} & X_G^{\gamma_4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & X_B^{\beta_1^B} & X_B^{\beta_2^B} \end{bmatrix}, \tag{A.100}$$

$$b := \begin{bmatrix} -A_{G5} + C_3 D_B + C_4 + C_5 D_B^{\gamma_1} + C_6 D_B^{\gamma_2} \\ C_3 D_B + \gamma_1 C_5 D_B^{\gamma_1} + \gamma_2 C_6 D_B^{\gamma_2} \\ -C_3 D_G - C_4 - C_5 D_G^{\gamma_3} - C_6 D_G^{\gamma_4} + (1 - \alpha_G \Xi_G) \left((1-\tau) D_G y_G - \alpha_G \Xi_G G_G^{unlev}(D_G) \right) + G_G(D_G) \\ -A_{B5} + (1 - \alpha_B \Xi_B) \left((1-\tau) D_B y_B - \alpha_B \Xi_B G_B^{unlev}(D_B) \right) + G_B(D_B) \\ -A_{B5} + Z\left(X_G\right) + B_4 \\ X_G Z'\left(X_G\right) \\ -A_{G5} + \hat{d}_G \left((s_G + 1) X_G \right) \\ -Z\left(X_B\right) + B_4 + \hat{d}_B \left((s_B + 1 - \dfrac{k_B / \Xi_B}{(1-\tau) X_{\bar{t}} y_B}) X_B \right) \end{bmatrix}, \tag{A.101}$$

$$C_5 = \frac{\bar{l}_3}{l_3} \left(\bar{A}_{G3}^{lev} - \alpha_B \Xi_B \bar{A}_{G3}^{unlev} \right), \tag{A.102}$$

and

$$C_6 = \frac{\bar{l}_4}{l_4} \left(\bar{A}_{G4}^{lev} - \alpha_B \Xi_B \bar{A}_{G4}^{unlev} \right). \tag{A.103}$$

The case with $X_G \geq X_B$:

The solution of the case $X_G \geq X_B$ is obtained immediately by renaming states in the solution of the presented case for $X_G < X_B$.

Alternative financing strategies:

Bankruptcy costs for any alternative financing strategy can be derived analogously by replacing the earnings levels at option exercise in Equations (A.88), (A.89), (A.96), and (A.97).

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