Q1.1 Show which of the following operators is Hermitian: (a) \( \frac{d}{dx} \), (b) \( i \frac{d}{dx} \), (c) \( 4(\frac{d^2}{dx^2}) \), (d) \( i(\frac{d^2}{dx^2}) \)

Q1.2 If \( \hat{T}_x = -(\hbar^2 / 2m) \frac{d^2}{dx^2} \) show that \( \langle T_x \rangle = \frac{\hbar^2}{2m} \int |\partial \Psi / \partial x|^2 d\tau \)

Q1.3 Let \( \hat{A} \) be an Hermitian operator. Show that \( \langle A^2 \rangle = \int |\hat{A}\psi|^2 d\tau \)

Q1.4 (a) If \( \hat{A} \) and \( \hat{B} \) are Hermitian operators, prove that their product \( \hat{A}\hat{B} \) is Hermitian if and only if \( \hat{A} \) and \( \hat{B} \) commute. (b) If \( \hat{A} \) and \( \hat{B} \) are Hermitian prove that \( \frac{1}{2}(\hat{A}\hat{B} + \hat{B}\hat{A}) \) is Hermitian. (c) Is \( \hat{x}\hat{p}_x \) Hermitian? (d) Is \( \frac{1}{2}(\hat{x}\hat{p}_x + \hat{p}_x\hat{x}) \) Hermitian?

Q1.5 (a) Show that the hydrogen-like wave functions \( 2p_x \) and \( 2p_1 \) are not orthogonal. (b) Use Schmidt orthogonalization to construct linear combinations of \( 2p_x \) and \( 2p_1 \) that are orthogonal. (c) Normalize your orthogonal linear combinations.

Q1.6 Extend the Schmidt orthogonalization procedure to the case of three fold degeneracy.

Q1.7 For a hydrogen atom in a \( p \) state, the possible outcomes of a measurement of \( L_z \) are \(-\hbar, 0, \) and \( \hbar \). For each of the following wave functions, give the probabilities of each of these three results: (a) \( 2p_z \), (b) \( 2p_y \), (c) \( 2p_1 \).

Q1.8 Suppose that a particle in a box of length \( L \) is in a non-stationary state with \( \Psi = 0 \) for \( x < L \) and for \( x > L \), and

\[
\Psi = \left( \frac{2}{L} \right)^{1/2} \left\{ \frac{1}{2} \exp\left[-(it/\hbar)h^2/8mL^2\right] \sin[\pi x/L] + \frac{\sqrt{3}}{2} e^{i\pi} \exp\left[-(it/\hbar)h^2/2mL^2\right] \sin[2\pi x/L] \right\}
\]

for \( 0 \leq x \leq L \). (a) Verify that \( \Psi \) is normalized. (b) If the energy is measured at time \( t \) give the possible outcomes and their probabilities. (c) Find \( \langle E \rangle \) at time \( t \). (d) Find \( \langle x \rangle \) at time \( t \). (e) Find the maximum and minimum values of \( \langle x \rangle \) as a function of time.

Q1.9 Suppose that a particle in a box of length \( L \) is in a non-stationary state with \( \Psi(x) = x \) for \( 0 \leq x \leq L/2 \) and \( \Psi(x) = L - x \) for \( L/2 \leq x \leq L \). (a) Normalize this wave function. (b) If the energy is measured for this state, list the possible outcomes and their probabilities.

Q1.10 (a) What is the value of \( \int_0^{\infty} f(x) \delta(x) dx \)? (b) Show that \( \int_{-\infty}^{\infty} |\delta(x - a)|^2 dx = \infty \)