CH651 Assignment 1, Due in Class, Wednesday September 30, 2015

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Q1.1 Show which of the following operators is Hermitian: (a) d/dx, (b) $i(d/dx)$, (c) $4(d^2/dx^2)$, (d) $i(d^2/dx^2)$

Q1.2 If $\hat{T}_x = -(\hbar^2/2m)d^2/dx^2$ show that $\langle T_x \rangle = (\hbar^2/2m)\int |\partial\Psi/\partial x|^2 d\tau$

Q1.3 Let $\hat{A}$ be an Hermitian operator. Show that $\langle A^2 \rangle = \int |\hat{A}\psi|^2 d\tau$

Q1.4 (a) If $\hat{A}$ and $\hat{B}$ are Hermitian operators, prove that their product $\hat{A}\hat{B}$ is Hermitian **if and only if** $\hat{A}$ and $\hat{B}$ commute. (b) If $\hat{A}$ and $\hat{B}$ are Hermitian prove that $\frac{1}{2}(\hat{A}\hat{B}+\hat{B}\hat{A})$ is Hermitian. (c) Is $\hat{x}\hat{p}_x$ Hermitian? (d) Is $\frac{1}{2}(\hat{x}\hat{p}_x + \hat{p}_x\hat{x})$ Hermitian?

Q1.5 (a) Show that the hydrogen-like wave functions $2p_x$ and $2p_1$ are **not** orthogonal. (b) Use Schmidt orthogonalization to construct linear combinations of $2p_x$ and $2p_1$ that are orthogonal. (c) Normalize your orthogonal linear combinations.

Q1.6 Extend the Schmidt orthogonalization procedure to the case of three fold degeneracy.

Q1.7 For a hydrogen atom in a p state, the possible outcomes of a measurement of L_z are $-\hbar$, 0, and $\hbar$. For each of the following wave functions, give the probabilities of each of these three results: (a) $2p_z$, (b) $2p_y$, (c) $2p_1$.

Q1.8 Suppose that a particle in a box of length L is in a non-stationary state with $\Psi = 0$ for $x < L$ and for $x > L$, and

$$\Psi = \left(\frac{2}{L}\right)^{1/2} \left\{\frac{1}{2}\exp[-(it/\hbar)h^2/8mL^2]\sin[\pi x/L] + \frac{\sqrt{3}}{2}e^{i\pi}\exp[-(it/\hbar)h^2/2mL^2]\sin[2\pi x/L]\right\}$$

for $0 \le x \le L$. (a) Verify that Ψ is normalized. (b) If the energy is measured at time t give the possible outcomes and their probabilities. (c) Find $\langle E \rangle$ at time t. (d) Find $\langle x \rangle$ at time t. (e) Find the maximum and minimum values of $\langle x \rangle$ as a function of time.

Q1.9 Suppose that a particle in a box of length L is in a non-stationary state with $\Psi(x) = x$ for $0 \le x \le L/2$ and $\Psi(x) = L - x$ for $L/2 \le x \le L$. (a) Normalize this wave function. (b) If the energy is measured for this state, list the possible outcomes and their probabilities.

Q1.10 (a) What is the value of $\int_0^\infty f(x)\delta(x)dx$? (b) Show that $\int_{-\infty}^{\infty} |\delta(x-a)|^2 dx = \infty$