Probability and Statistics (and Integrals)

1. Suppose we are given the following data:

x	P(x)
1	0.20
2	0.25
4	0.55

(a) Calculate the average value of x, <x>.
(b) Calculate the average value of x^2, <x^2>.

2. Calculate the following integrals:

a. $\int_{-A}^{A} f_{odd}(x)dx$

b. $\int_{-A}^{A} f_{odd}(x) f_{even}(x)dx$

3. Prove explicitly that $\int_{-\infty}^{\infty} e^{-ax^2}dx = 2\int_{-\infty}^{0} e^{-ax^2}dx$

4. Show explicitly that $\int_{-\infty}^{\infty} xe^{-ax^2}dx = 0$

5. Without calculators or computers, evaluate the integral $\int_{4}^{20} xe^{x^2}dx$. (Hint: substitution)

6. Without calculators or computers, evaluate the integrals. (Hint: integration by parts)

a. $\int_{A}^{B} xe^{-6x}dx$

b. $\int (4x+1)\cos(\pi x)dx$

c. $\int \ln(x)dx$

Series and Limits

1. Show that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$. When is this approximation, $e^x \approx 1 + x$, reasonable?
2. Prove that $\int e^{ax}dx = \frac{1}{a}e^{ax} + C$ by using the Taylor series expansion of e^{ax}.
3. What is the geometric series?
4. Evaluate the series $S = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$

The Binomial Distribution and the Stirling Approximation

1. Calculate the number of ways of dividing 10 distinguishable objects into three groups containing 2, 5, and 3 objects.
2. In how many ways can a committee of three be chosen from nine people?
3. Prove that $x\ln(x) \rightarrow 0$ as $x \rightarrow 0$.
4. Prove that the maximum value of $W(N,N_1) = N!/(N-N_1!)N_1!$ is given by $N_1 = N/2$.
5. Prove that the maximum value of $W(N_1,N_2,\ldots N_r) = \frac{N!}{N_1!N_2!\ldots N_r!}$, where $N_1+N_2+\ldots N_r=N$ is given by $N_1=N_2=\ldots=N_r=N/r$. Plot your data to support your answer.
6. Prove that $\sum_{k=0}^{N} \frac{N!}{k!(N-k)!} = 2^N$