Discussion Section #3 – February 7, 2014
(Chapters 1 – 5)

1. **Calculating the entropy of mixing**. Consider a lattice with N sites and n green particles. Consider another lattice, adjacent to the first, with M sites and m red particles. Assume that the green and red particles cannot switch lattices. This is state A.

(a) What is the total number of configurations W_A of the system in state A?

(b) Now assume that all $N+M$ sites are available to all the green and red particles. The particles remain distinguishable by their color. This is state B. Now what is the total number of configurations W_B of the system?

Now take $N=M$ and $n=m$ for the following two problems.
(c) Using Stirling's approximation, what is the ratio of W_A/W_B?

(d) Which state, A or B, has the greatest entropy? Calculate the entropy difference given by

$$\Delta S = S_A - S_B = -k\ln\left(\frac{W_A}{W_B}\right).$$

2. Consider two systems, A and B. System A has N_A=10 particles and system B has N_B=4 particles. Each particle has two possible energies, *ε=0 or ε=1*. Suppose that system A starts with n_A=2 particles with *ε=1* and (N_A - n_A) particles with *ε=0* (so system A energy U_A=2) and system B starts with n_B=2 particles with *ε=1* and (N_B – n_B) particles with *ε=0* (system B energy U_B=2 also). This situation is shown in Figure 2b. The two systems are brought in thermal contact and the total energy $U=U_A+U_B$ is conserved.

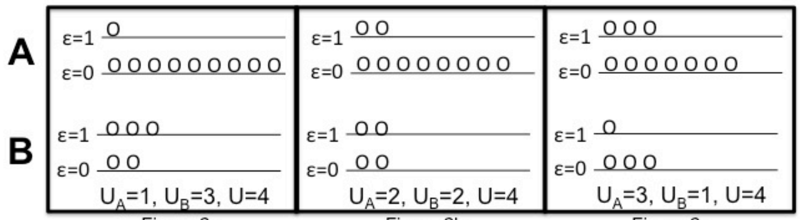

Figure 2a Figure 2b Figure 2c

(i) Let *W* be the multiplicity of the total system (system A + system B)
(ii) Let W_A and W_B be the multiplicities representing systems A and B respectively.
(iii) Let p_0 and p_1 be the probabilities of finding particles with energies *ε=0 and ε=1* respectively. In general there will be different distributions of these probabilities for systems A and B. Our goal is to understand what happens to these probability distributions when systems A and B are brought into thermal contact with one another but the total energy stays fixed because the total system (A+B) is thermally isolated from its surroundings.

(a) Calculate all the indicated unspecified quantities below for state a (represented by Figure 2a), state b (Figure 2b), and state c (Figure 2c).

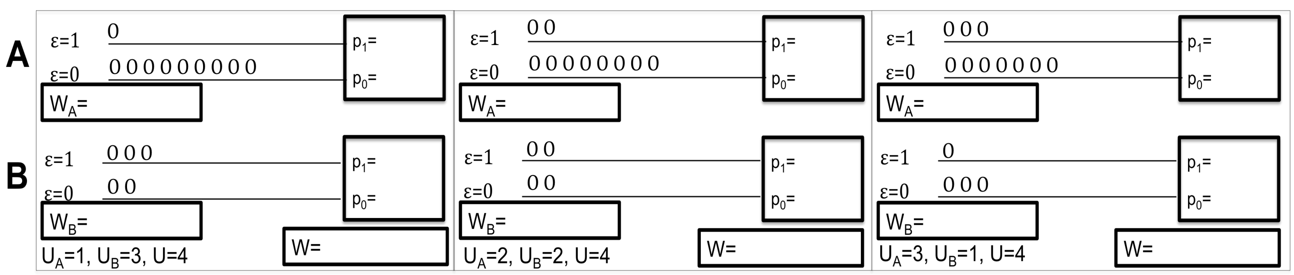

Figure 2a Figure 2b Figure 2c

(b) What is the total multiplicity of the initial state represented in Figure 2b? Which state would represent the final thermalized state? In this thermalized state, Is the energy of system A equal to the energy of system B?

The tendency of heat to flow is not always a tendency to equalize energies. It is a tendency to maximize multiplicity. We will see later that the concept of temperature describes the driving force for energy exchange. The tendency toward maximum multiplicity is a tendency toward equal temperatures, not equal energies.

(c) Examining the probability distributions of particles of each energy level of all three states, why does it make sense that the answer to (b) is the final state?

3. Lets see if we can prove something general from the observations above using the concept of maximizing multiplicity of the total constant energy system.

(a) Since there is only exchange of energy and no exchange of particles between systems A and B (i.e. N_A and N_B are fixed), W_A is a function only of the variable n_A and W_B is a function only of the variable n_B, the numbers of particles in the *ε=1* level in systems A and B respectively. The total multiplicity $W(n_A,n_B)=W_A(n_A)W_B(n_B)$ is a multidimensional function. Write down an expression for lnW and its total differential d lnW in terms of variations dn_A and dn_B.
(b) Write down an expression for the total energy of the system U in terms of n_A and n_B give its total differential dU. If U is constant what does this imply about dn_A and dn_B?
(c) Use your last finding in (b) to simplify your expression for d lnW from (a).
(d) Set your simplified expression for d lnW from (c) equal to zero to maximize lnW (and hence maximize W) and then use Stirling's approximation to obtain a relationship between the numbers of particles in the different levels in the different systems when they reach thermal equilibrium (i.e. them maximize the total entropy (multiplicity).

(e) Explain how this is consistent with your findings from 2.