Discussion Section #2 – January 31, 2014

Additional comments from Professor Coker:

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Level 1			
Level 2			
Level 3			
Level 4			
Level 5			
Level 6			

(What is the probability that the ink is diffused up to Level 1? Level 2? … Level 6? Relate this model problem to diffusion of molecules in a room.)

Questions for Discussion

1. **De-mixing is improbable.** Suppose that you have 2V black particles and 2V white particles in 4V lattice sites. There are 2V lattice sites on the left and 2V lattice sites on the right, separated by a permeable wall. The total volume is fixed. Show that perfect de-mixing (all white on one side, all black on the other) becomes increasingly improbable as V increases. (Hint: refer to Example 2.3 in the book).

2. **Maximum of binomial distribution.** Find the value *n*=*n** that causes the function

$$W = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n}$$

to be a maximum, for constants *p* and *N*. Use Stirling's approximation $x! \approx (x/e)^x$. Note that it is easier to find the value of *n* that maximizes lnW than the value that maximizes W. The value of *n** will be the same.

3. **Winter in Boston**. Snow storms in Boston are either light (L) or heavy (H) with typical accumulation in a light storm a_L=6 inches, and in heavy storm, a_H=12 inches. The average snow accumulation in a winter storm is $\langle a \rangle$=7 inches. Compute the probability of a heavy snowstorm during a Boston winter.

4. Consider two identical regions of space initially separated by a removable wall. Suppose the left region is represented by *M* lattice sites and the right region is represented by an additional *M* sites. Initially, with the separating wall in place, the left region contains *N* particles, while the right region contains no particles.

(i) Write down an expression for the total number of configurations, *W*, for this system.

(ii) If the entropy, *S*, of the system is related to W by $S=k_B lnW$, where k_B is the Boltzmann constant, using Stirling's approximation, calculate the entropy S_i of the initial state described above.

(iii) Now the wall is removed and the system equilibrates to its final state where its entropy is S_f. Calculate S_f.

(iv) Compute the change in entropy ΔS. Simplify ΔS by assuming the ideal gas limit M>>N so that 1-N/M≈1.