CH352 Assignment 3. Due: Friday, March 28, 2014.

Prof. David F. Coker

Q1 Adiabatic quasi static volume and temperature changes in ideal gases. In the last assignment you showed that the internal energy of a monatomic ideal gas sample containing N particles was proportional to the temperature, T and N, so that $U = \frac{3}{2}Nk_BT$. Since the heat capacity at constant volume is $C_V = (\partial U/\partial T)_{V,N}$, $C_V = \frac{3}{2}Nk_B$ for an ideal gas. Thus, in this system, we find $dU = C_V dT$.

(i) Use the above result in the differential form of the fundamental energy equation, $dU = TdS - pdV$, (assuming fixed N) to obtain an expression for the differential entropy change, dS in general, constant N, ideal gas processes.

(ii) Consider a quasi static transformation of an N particle sample of idea gas and re-express your result for dS from (i) for this special case ideal gas process.

(iii) Integrate your result from (ii) to obtain a general expression for the entropy change $\Delta S = S_2 - S_1$ accompanying the quasi static fixed N transformation from (T_1, V_1), to (T_2, V_2).

(iv) In an adiabatic quasi-static expansion of an ideal gas, how do you reconcile the following two facts: (1) the increase in volume should lead to an increase in entropy, but (2) in an adiabatic process $\delta q = 0$ so there should be no change in entropy (since $dS = \delta q/T = 0$)?

Q2 Enthalpy of vaporization of water at $T = 473$K. Suppose you want to know how much heat it would take to boil water at $p = 1$ atm. and $T = 473$K, rather than at $T = 373$K. At $T = 373$K the enthalpy of vaporization is $\Delta H_{\text{boil}}(T = 373\text{K}) = 40.7$ kJ mol^{-1}. Assuming that the heat capacities of the liquid ($C_{p,\text{liquid}} = 75$ J K^{-1} mol^{-1}) and the vapor ($C_{p,\text{vapor}} = 3.5$ J K^{-1} mol^{-1}) are constant over this temperature range, calculate the enthalpy of vaporization at $T = 473$K, $\Delta H_{\text{boil}}(T = 473\text{K})$.

Q3 The work of compression. One mole of a van der Waals gas is compressed quasi-statically and isothermally from V_1 to V_2. For a van der Waals gas the equation of state is

$$p = \frac{RT}{(V-b)} - \frac{a}{V^2} \tag{1}$$

where a and b are material constants, V, is the volume, and RT is the gas constant times temperature.

(i) Write an expression for the work done.

(ii) Is more or less work required in the low density limit than for an ideal gas? What about the high density limit? Why?

Q4 Ideal efficiency of a car engine. Suppose the compression ratio in your car engine is $V_2/V_1 = 8$. For a diatomic gas, $C_V = (5/2)Nk_B$ and for ethane $C_V \sim 5Nk_B$.

(i) What is the efficiency of your engine if the working fluid is a diatomic gas?

(ii) Which is more efficient: a diatomic gas or ethane?

(iii) Would your engine be more or less efficient with a higher compression ratio?

Q5 Computing enthalpy and entropy with a temperature dependent heat capacity. The heat capacity of liquid n-butane depends on temperature:

$$C_p(T) = a + bT \tag{2}$$

where $a = 100$ J K^{-1} mol^{-1} and $b = 0.1067$ J K^{-2} mol^{-1} from its freezing temperature $T_f = 140$K to $T_b = 270$K, its boiling temperature.

(i) Compute ΔH for heating liquid butane from $T_A = 170$K to $T_B = 270$K.

(ii) Compute ΔS for the same process.

Q6 Objects in thermal contact. Suppose two objects A and B with heat capacities C_A and C_B and initial temperatures T_A and T_B, are brought in to thermal contact. If $C_A >> C_B$ is the equilibrium T closer to T_A or T_B?

Q7 Heating a house. If your living room, having volume 6m × 6m × 3m $\sim$ 100m^3, were perfectly insulated how much energy would be needed to raise the temperature inside the room from $T_{in} = 0$°C to $T_{fin} = 25$°C? (Note that $C_V = C_P - nR$, where n is the number of moles of ideal gas.)