Quiz 8

Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page.

Question:	1	2	3	Total
Points:	10	15	0	25
Score:				

Name: ______________________________

1. (10 points) The energy change of a system with a single component can be expressed as

$$\mathrm{d}U = T\,\mathrm{d}S - p\,\mathrm{d}V + \mu\,\mathrm{d}N \tag{1}$$

where each of the variables T, p, and μ are partial derivatives of the internal energy U of the system. What are the partial derivative expressions for these three variables?

Solution:

$$T = \left(\frac{\partial U}{\partial S}\right)_{V,N}$$
$$-p = \left(\frac{\partial U}{\partial V}\right)_{S,N}$$
$$\mu = \left(\frac{\partial U}{\partial N}\right)_{S,V}$$

2. Suppose we have a system with two parts A and B with fixed volume and number that are brought into thermal contact. The system is isolated from its surroundings, energy is allowed to flow between the two parts, but the parts are otherwise separated.

(a) (5 points) What do we know about the change in the internal energy $\mathrm{d}U$ of the system as it approaches equilibrium? Please circle one.

Solution: The system is **isolated** so $\boxed{\mathrm{d}U = 0}$.

(b) (5 points) What do we know about the change in the entropy $\mathrm{d}S$ of the system as it approaches equilibrium? Please circle one.

Solution: Entropy always increases or remains constant for isolated systems, so $\boxed{\mathrm{d}S \geq 0}$.

(c) (5 points) The relationship between what two variables will tell us which direction energy will flow? Please circle one.

Solution: As shown in class, the relationship between $\boxed{T_A \text{ and } T_B}$ determines the direction of energy flow.

3. For fun if you finish early: Because U is a first order homogenous function, we showed in class that

$$U = TS - pV + \sum_i \mu_i N_i. \tag{2}$$

Take the total derivative of this function to derive the Gibbs-Duhem equation:

$$0 = S\,\mathrm{d}T - V\,\mathrm{d}p + \sum_i N_i\,\mathrm{d}\mu_i. \tag{3}$$

Solution:

$$\mathrm{d}U = T\,\mathrm{d}S + S\,\mathrm{d}T - p\,\mathrm{d}V - V\,\mathrm{d}p + \sum_i \left(N_i\,\mathrm{d}\mu_i + \mu_i\,\mathrm{d}N_i\right)$$

but we also know

$$\begin{aligned} \mathrm{d}U &= T\,\mathrm{d}S - p\,\mathrm{d}V + \mu\,\mathrm{d}N \\ &\therefore \\ &\boxed{0 = S\,\mathrm{d}T - V\,\mathrm{d}p + \sum_i N_i\,\mathrm{d}\mu_i} \end{aligned}$$

This result is called the Gibbs-Duhem equation.