Quiz 7

Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page.

| Question: | 1 | 2 | Total |
|---|---|---|---|
| Points: | 25 | 0 | 25 |
| Score: | | | |

Name: _____

1. Suppose we have a molecule with two angles $\theta$ and $\phi$. The energy with respect to $\theta$ is $\epsilon_\theta(\theta) = 1-\cos(\theta-\alpha)$, and the energy with respect to $\phi$ is $\epsilon_\phi(\phi) = 1 - \cos(2\phi - \beta)$ where $\alpha$ and $\beta$ are real constants.

   (a) (10 points) What is an absolute minimum of the total energy $E(\theta, \phi) = \epsilon_\theta(\theta) + \epsilon_\phi(\phi)$ and at what angles does this minimum occur? Note, because this is a periodic function, it has an infinite number of minima, but just find one.

   $E_{\text{min}} =$

   $\theta_{\text{min}} =$

   $\phi_{\text{min}} =$

   (b) (5 points) Now Taylor expand this energy function around some point ($\theta = \alpha, \phi = \beta/2$) to the first non-vanishing order (the lowest order that does not make the function zero everywhere).

   $E(\theta, \phi) \approx$

   (c) (10 points) Someone else has found a Taylor series for a different energy function:

   $$E(x, y) \approx (x + a)^2 + (y + b)^2.$$

   If the angles $x$ and $y$ are constrained such that $x + y = m$, what is the minimum energy and the angles at which it occurs for this constrained system?

   $E_{\text{min}} =$

   $x_{\text{min}} =$

   $y_{\text{min}} =$

2. For fun if you finish early: Let's impose essentially the same constraint on our initial energy function from (a) and (b):

$$\theta + \phi = m.$$

What is the energy minimum of the Taylor series approximation? Can you find the constrained energy minimum without doing a Taylor series expansion?