Quiz 5

Question:	1	2	Total
Points:	25	0	25
Score:			

Name and section: _____

1. The general form of the Taylor series of a function $f(x)$ around the point a is

$$f(x) = \sum_{i=0}^{\infty} \frac{1}{i!} \left. \frac{d^i f(x)}{dx^i} \right|_{x=a} (x-a)^i \tag{1}$$

where the value of the ith derivative is evaluated at the point $x = a$.

(a) (10 points) What is the Taylor series of $f(x) = \sin(x)$ up to $i = 3$ (third order) using the above expression around the point $x = 0$?

$$f(0) = 0$$
$$f'(0) = 1$$
$$f''(0) = 0$$
$$f^{g(3)} = -1$$
$$f(x) \approx 0 + 1x + \frac{1}{2}0\,x^2 - \frac{1}{6}1\,x^3$$
$$\boxed{f(x) \approx x - \frac{1}{6}x^3}$$

(b) (10 points) What is the Taylor series of $g(x) = ax^3 - bx$ around $x = 0$ up to third order $(i = 3)$?

$$g(0) = 0$$
$$g'(0) = -b$$
$$g''(0) = 0$$
$$g^{(3)} = 6a$$
$$\boxed{g(x) = ax^3 - bx}$$

(c) (5 points) How does the Taylor series of $g(x)$ change if we take the Taylor series to fourth order $(i = 4)$? Answer in no more than one sentence.

It doesn't change to any higher orders.

2. For fun if you finish early: Up to second order, what is the Taylor series of the function

$$h(x, y) = \sin(x) \cos(y) \tag{2}$$

around the point $(x, y) = (\pi, 0)$?