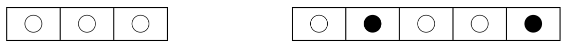
Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page.

Question:	1	2	Total
Points:	25	0	25
Score:			

Name and section: ______________________

1. Consider a system with three white molecules contained in a flexible membrane. This membrane and particles are in a bath of black molecules. The membrane can stretch, and it is permeable to black molecules, but the white molecules cannot pass through. For example, below are both valid configurations of the system inside the membrane:



   (a) (10 points) What are the multiplicities when the membrane has three (e.g., above left), four, five (e.g., above right), and six molecules inside? Ignore the configuration of the bath of black particles.

$$W_3 = \binom{3}{3} = 1$$

$$W_4 = \binom{4}{3} = 4$$

$$W_5 = \binom{5}{3} = 10$$

$$W_6 = \binom{6}{3} = 20$$

   (b) (10 points) If the membrane can only stretch to fit a maximum of six molecules inside, but otherwise all configurations are equally likely, what are the probabilities of the membrane containing three, four, five, or six molecules?

$$p_3 = \tfrac{1}{35}$$

$$p_4 = \tfrac{4}{35}$$

$$p_5 = \tfrac{2}{7}$$

$$p_6 = \tfrac{4}{7}$$

   (c) (5 points) Based on your probabilities above, what is the expectation value of the number of molecules inside the membrane, $\langle n \rangle$? Please feel free to leave the answer as a fraction.

$$\begin{aligned}\langle n \rangle &= 3\frac{1}{35} + 4\frac{4}{35} + 5\frac{10}{35} + 6\frac{20}{35} \\ &= \frac{189}{35} = \frac{27}{5} = 5.4\end{aligned}$$

2. For fun if you finish early: Real membranes similar to that described above are actually used for membrane/buffer exchanges for proteins and biomolecules. Say for example, we have three black proteins inside the membrane as above, but we also start with two white salt molecules inside the membrane, but the salt can move through the membrane. Now we want to switch out the white salt with a $\times$ salt, so we put the membrane in a bath of $\times$ salt. Finally, let's also say that the membrane will always contain five molecules.

   (a) If we consider only the configurations of the system inside the membrane, give a permutation of the most likely configuration.

●	●	●	○	$\times$

   (b) How does the most likely configuration change if we instead do consider the configurations of the bath?

   Now including the bath, the greatest entropy (and highest multiplicity) for the whole system will be when both white salt molecules are in the bath and the membrane contains two $\times$s.