Quiz 2

Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page.

Question:	1	2	3	Total
Points:	15	10	0	25
Score:				

Name and section: _____

1. Given that you have a coin that you flip N times in independent trials, answer the following questions.
   (a) (5 points) If the coin is fair ($p_H = p_T = \frac{1}{2}$), what is the probability of getting exactly seven heads in the N trials ($N \geq 7$)?

$$p = \left(\frac{1}{2}\right)^N \binom{N}{7} = \left(\frac{1}{2}\right)^N \frac{N!}{7!(N-7)!}$$

   (b) (10 points) For the same fair coin, if $N = 20$ and given that the first 10 flips are all heads, what will be the average total number of heads $\langle n_{h,\text{total}} \rangle$ in the 20 coin flips?
   The first key is to recognize that if the first 10 flips are all heads, then we are really only dealing with 10 random trials. For 10 flips of a fair coin, $\langle n_h \rangle = 5$. Since we already had 10 heads on the first 10 flips, $\langle n_{h,\text{total}} \rangle = 10 + 5 = 15$.

2. (10 points) You have two dice, each with four sides (with the sides numbered 1-4), and you roll both and add up the number shown on each die. What is the average of the sum?
   The lowest possible value is 2 (if you roll two 1s) and the greatest is 8, so we can sum over the possibilities:

$$\begin{aligned}\langle \text{sum} \rangle &= \sum_{i=2}^{8} i\, p(i) \\ &= 2 \times \frac{1}{16} + 3 \times \frac{2}{16} + 4 \times \frac{3}{16} + 5 \times \frac{4}{16} + 6 \times \frac{3}{16} + 7 \times \frac{2}{16} + 8 \times \frac{1}{16} \\ \langle \text{sum} \rangle &= 5\end{aligned}$$

   Alternatively, you could realize that this distribution is symmetric, so the middle number (5) is the average.
   This is essentially the problem we did in the first discussion except with 4-sided dice.

3. For fun if you finish early: A circular table has 60 chairs around it. There are N people seated at this table so that the next person seated must sit next to someone. Find the smallest possible value of N. (AHSME 1991 #15 via `https://ocw.mit.edu/high-school/mathematics/combinatorics-the-fine-art-of-counting/assignments/`)

   $N_{\min} = 20$ because there can be two empty seats between every person so the people would be sitting every three seats. See the website for a fuller explanation.