Quiz 13

Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page.

Question:	1	2	Total
Points:	20	5	25
Score:			

Name: ______________________________

1. Consider a system with three energy levels as described in the table below. Assume the system has Boltzmann-weighted probabilities ($p(\epsilon) \propto \mathrm{e}^{-\epsilon\beta}$).

ϵ	g
0	1
ϵ_0	1
$2\epsilon_0$	γ

(a) (5 points) Write an expression for the partition function q as a function of energy, degeneracy, and temperature.

Solution:

$$q = 1 + \mathrm{e}^{-\epsilon_0\beta} + \gamma\mathrm{e}^{-2\epsilon_0\beta}$$

(b) (5 points) What is an expression for the average energy? This can be left as a (simplified) function of q.

Solution:

$$\langle\epsilon\rangle = \frac{1}{q}\left(\epsilon_0\mathrm{e}^{-\epsilon_0\beta} + 2\epsilon_0\gamma\mathrm{e}^{-2\epsilon_0\beta}\right)$$

(c) (5 points) At what temperature will the probabilities of the first and third energy levels be the same (i.e., $p_1^* = p_3^*$). T will be a function of ϵ_0 and γ.

Solution:

$$\frac{1}{q} = \frac{1}{q}\gamma\mathrm{e}^{-2\epsilon_0\beta}$$

$$\frac{1}{\gamma} = \mathrm{e}^{-2\epsilon_0\beta}$$

$$\ln\gamma = 2\epsilon_0\beta$$

$$\beta = \frac{\ln\gamma}{2\epsilon_0}$$

$$\boxed{T = \frac{2\epsilon_0}{k_B\ln\gamma}}$$

(d) (5 points) For $\epsilon_0 = \ln(2)/\beta$ and $\gamma = 2$, compute the equilibrium probabilities of the three energy levels.

Solution:

$$\begin{aligned} q &= 1 + \frac{1}{2} + 2\frac{1}{4} = 2 \\ p_1^* &= \frac{1}{2} \\ p_2^* &= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \\ p_3^* &= 2 \cdot \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{4} \end{aligned}$$

2. (5 points) For fun if you have time: What could Thomas as the teaching fellow for the course have done better?