Quiz 11

Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page.

Question:	1	2	3	Total
Points:	13	12	0	25
Score:				

Name: ______________________________

1. Legendre transforms

   (a) (5 points) In going from internal energy $U(S,V,N)$ to Helmholtz free energy $F(T,V,N)$, we can find F as $F = U - TS$. What is a similar expression to go from Helmholtz free energy $F(T,V,N)$ to Gibbs free energy $G(T,p,N)$?

   Solution: We need to go from V to p, so

   $$G = F + pV$$

   (b) (8 points) Enthalpy H is $H(S,p,N) = U + pV$. What is an expression for $\mathrm{d}H$ in terms of S, p, and N for a single component system?

   Solution:

   $$\begin{aligned} \mathrm{d}H &= \mathrm{d}U + p\,\mathrm{d}V + V\,\mathrm{d}p \\ &= T\,\mathrm{d}S - p\,\mathrm{d}V + \mu\,\mathrm{d}N + p\,\mathrm{d}V + V\,\mathrm{d}p \\ &\boxed{\mathrm{d}H = T\,\mathrm{d}S + V\,\mathrm{d}p + \mu\,\mathrm{d}N} \end{aligned}$$

2. (12 points) Partial derivative expressions

   Here, we are looking for partial derivatives of energy terms. For example,

   $$T = \left(\frac{\partial U}{\partial S}\right)_{V,N} \tag{1}$$

   is an expression for T as a partial derivative of the internal energy U with V and N held constant.

   (a) What is an expression for p as a derivative of F?

   Solution:

   $$p = -\left(\frac{\partial F}{\partial V}\right)_{T,N}$$

   (b) S as a derivative of F?

   Solution:

   $$S = -\left(\frac{\partial F}{\partial T}\right)_{V,N}$$

(c) V as a derivative of H?

Solution:

$$V = \left(\frac{\partial H}{\partial p}\right)_{S,N}$$

3. For fun if you have time: In class we derived a Maxwell relation from $U(S, V)$:

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V .$$

Again assuming constant N, what Maxwell relation can be derived from $H(S, p)$? $F(T, V)$?

Solution:

$$\frac{\partial^2 H}{\partial S \partial p} = \frac{\partial^2 H}{\partial p \partial S}$$
$$\left(\frac{\partial V}{\partial S}\right)_p = \left(\frac{\partial T}{\partial p}\right)_S$$

$$\frac{\partial^2 F}{\partial T \partial V} = \frac{\partial^2 F}{\partial V \partial T}$$
$$-\left(\frac{\partial p}{\partial T}\right)_V = -\left(\frac{\partial S}{\partial V}\right)_T$$
$$\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$