Quiz 10

Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page.

Question:	1	Total
Points:	25	25
Score:		

Name: ____

A general partition function Z for a Boltzmann distribution with discrete energy states is

$$Z = \sum_i \mathrm{e}^{-\beta E_i}$$

where $\beta = (k_B T)^{-1}$, E_i is the energy of microstate i, and the sum is over all possible microstates.

1. Consider a gas of N carbon monoxide (CO) molecules above a metallic surface. Treat each CO molecule as an independent, distinguishable two level system. Assume each CO molecule can exist in one of two energy states: free in the gas ($E_1 = 0$) and bound to the surface ($E_2 = -\epsilon$). Further assume there is a multiplicity of gas states γ_{g} and multiplicity of bound states γ_{b} available to each molecule, and that $\gamma_{\mathrm{g}} > \gamma_{\mathrm{b}} \gg N$.

   (a) (10 points) What is the partition function $q(T)$ for a single CO molecule in terms of ϵ, γ_{g}, γ_{b}, and $k_B T$?

   $q(T) =$

   (b) (10 points) What is the partition function $Q(T)$ for N molecules of CO in terms of N, ϵ, γ_{g}, γ_{b}, and $k_B T$? Be sure to account for the indistinguishability of the CO molecules.

   $Q(T) =$

   (c) (5 points) What is the average energy, $U = \langle E \rangle$ for N molecules of CO in terms of N, ϵ, γ_{g}, γ_{b}, and $k_B T$?

   $U(T) =$

For fun, if you have time:

(d) What is the constant volume heat capacity, $C_V = \left(\frac{\partial U}{\partial T}\right)_V$ for N molecules of CO in terms of N, ϵ, γ_{g}, γ_{b}, and $k_B T$.

$C_V(T) =$

(e) Plot the energy $U(T)$ and constant volume heat capacity $C_V(T)$ as a function of temperature. Clearly indicate the high and low ($T = 0$) temperature limits.