A general partition function $Z$ for a Boltzmann distribution with discrete energy states is

$$Z = \sum_i e^{-\beta E_i},$$

where $\beta = (k_B T)^{-1}$, $E_i$ is the energy of microstate $i$, and the sum is over all possible microstates.

1. Consider a gas of $N$ carbon monoxide (CO) molecules above a metallic surface. Treat each CO molecule as an independent, distinguishable two level system. Assume each CO molecule can exist in one of two energy states: free in the gas ($E_1 = 0$) and bound to the surface ($E_2 = -\epsilon$). Further assume there is a multiplicity of gas states $\gamma_g$ and multiplicity of bound states $\gamma_b$ available to each molecule, and that $\gamma_g > \gamma_b \gg N$.

(a) (10 points) What is the partition function $q(T)$ for a single CO molecule in terms of $\epsilon, \gamma_g, \gamma_b,$ and $k_B T$?

$$q(T) =$$

(b) (10 points) What is the partition function $Q(T)$ for $N$ molecules of CO in terms of $N, \epsilon, \gamma_g, \gamma_b$, and $k_B T$? Be sure to account for the indistinguishability of the CO molecules.

$$Q(T) =$$

(c) (5 points) What is the average energy, $U = \langle E \rangle$ for $N$ molecules of CO in terms of $N, \epsilon, \gamma_g, \gamma_b$, and $k_B T$?

$$U(T) =$$
For fun, if you have time:

(d) What is the constant volume heat capacity, $C_V = \left( \frac{\partial U}{\partial T} \right)_V$ for $N$ molecules of CO in terms of $N$, $\epsilon$, $\gamma_\xi$, $\gamma_b$, and $k_B T$.

$$C_V(T) =$$

(e) Plot the energy $U(T)$ and constant volume heat capacity $C_V(T)$ as a function of temperature. Clearly indicate the high and low ($T = 0$) temperature limits.