Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page.

Question:	1	2	3	Total
Points:	15	10	0	25
Score:				

Name and section: _____

1. Given that you have a coin that you flip four times in independent trials, answer the following questions.
   (a) (5 points) If the coin is fair ($p_H = p_T = \frac{1}{2}$), what is the probability of getting the sequence $HHTH$?
   $$\left(\tfrac{1}{2}\right)^4 = \tfrac{1}{16}$$
   (b) (5 points) For the same fair coin, what is the probability of the sequence $TTHH$?
   $$\left(\tfrac{1}{2}\right)^4 = \tfrac{1}{16}$$
   (c) (5 points) If instead of a fair coin, $p_H = \frac{3}{4}$, what is the probability of the sequence $HHTH$? Please leave the answer as a fraction.
   $$\left(\tfrac{3}{4}\right)^3 \times \left(1 - \tfrac{3}{4}\right) = \tfrac{27}{256}$$

2. Assume that having a male or a female child are mutually exclusive, collectively exhaustive, and equally likely. Also assume that a child's gender is independent of their siblings' gender.
   (a) (5 points) What is the likelihood of having two boys and a girl, in any order? Be sure to consider all the different ways this could happen.
   $$\left(\tfrac{1}{2}\right)^3 \times \binom{3}{2} = \tfrac{3}{8}$$
   (b) (5 points) What is the probability of having two boys and a girl or a boy and two girls, in any order?
   $$\tfrac{3}{8} + \tfrac{3}{8} = \tfrac{6}{8} = \tfrac{3}{4}$$

3. For fun if you finish early: If you have paper with parallel lines a distance l apart, what is the probability that a needle of length l dropped on the paper will cross one of those lines?

   First, we can simplify this somewhat by realizing that we can look at only one area between two lines on the paper. Also, assuming no edge effects (getting off the edge of the page), we only need to worry about one dimension. The probability of landing anywhere between two lines is equal with probability $p(x) = \frac{1}{l}$ of landing at x. If one end of the needle lands at x, then it will cross a line if its length projected onto the x-axis is x or longer:

$$p_{\text{cross}} = \int_0^l p(x) p_l(x)\, \mathrm{d}x \tag{1}$$

   where $p_l(x)$ is the probability that the projected length of the needle will be x or longer.

   We know $p(x) = \frac{1}{l}$, so we just need to find $p_l(x)$. The length of the needle projected onto the x-axis is just $l \sin\theta$ where θ is the angle the needle makes the parallel lines. Then, to find the probability $p_l(x)$, we can integrate from the minimum possible angle to larger angles and divide by all possible angles. I do this just in the region $[0, \frac{\pi}{2}]$ because it is symmetric.

$$p_l(x) = \frac{\int_{\arcsin(x/l)}^{\pi/2} \mathrm{d}\theta}{\int_0^{\pi/2} \mathrm{d}\theta} \tag{2}$$

$$= 1 - \frac{2}{\pi} \arcsin\frac{x}{l} \tag{3}$$

   Putting this back into eq. (1):

$$p_{\text{cross}} = \int_0^l \frac{1}{l} \left(1 - \frac{2}{\pi} \arcsin\frac{x}{l}\right) \mathrm{d}x \tag{4}$$

$$p_{\text{cross}} = \frac{2}{\pi} \tag{5}$$