

Name: _____

Answer the questions in the spaces provided. If you run out of room for showing work, continue on the back of the page, but please write your final answer in the box.

All work should be completed on your own, and all students are expected to comply with the BU Academic Code of Conduct.

You will not need a calculator to complete this exam, and you will likely find the equation sheet at the end helpful.

Question	Points	Score
1	25	
2	25	
3	25	
4	25	
Total:	100	

1. (25 points) Robert Brown placed pollen grains of mass, m , in a cylinder of water at temperature, T , and measured their heights, h , above the bottom of the cylinder. The distribution of heights is given by

$$p(h) = C \exp(-mgh/k_B T) \quad (1)$$

where g is the gravitational acceleration constant, k_B is Boltzmann's constant, and C is a normalization constant. The height, h , varies from 0 to ∞ .

- (a) Determine the value of the normalization constant, C . It will be a function of m , g , T , and k_B .

$C =$

- (b) Determine the average height of the pollen grains h in terms of m , g , T , and k_B .

$\langle h \rangle =$

- (c) Determine the variance σ_h^2 of the distribution function, defined as $\langle (h - \langle h \rangle)^2 \rangle$, in terms of m , g , T , and k_B .

$$\sigma_h^2 =$$

- (d) If the temperature is increased by 10%, what will be the percent change in the average height of the pollen grains?

$$\Delta \langle h \rangle = \frac{\langle h_f \rangle - \langle h_i \rangle}{\langle h_i \rangle} \times 100\% \quad (2)$$

$$\Delta \langle h \rangle = \quad \%$$

3. (25 points) Consider a diatomic molecule with atoms bound together by a Morse potential:

$$V(r) = D \left(1 - e^{-\alpha(r-r_e)} \right)^2 \quad (3)$$

where D , α , and r_e are constants and r is the distance between the two nuclei.

- (a) What are the point r_{\min} where the minimum of this function occurs and the value V_{\min} of the potential at that point?

$$r_{\min} =$$

$$V_{\min} =$$

- (b) Now, to get an understanding of the behavior near the minimum of the potential, Taylor expand the potential around the minimum point r_{\min} to second order (up to r^2 , that is).

$$V(r) \approx$$

- (c) Up to second order we obtain a quadratic expression, which can be thought of as a simple harmonic oscillator potential:

$$V_{\text{SHO}} = \frac{1}{2}kx^2 \quad (4)$$

where x is the displacement from the potential minimum and k is the spring constant. From classical physics, we know that the frequency of a harmonic oscillator is $\omega = \sqrt{\frac{k}{\mu}}$ where μ is the reduced mass of the system. Give an expression for the frequency ω of this system in terms of μ , D , and α assuming the second order Taylor series is sufficiently accurate.

$\omega =$

4. (25 points) Consider a one-dimensional lattice. There are $N/2$ lattice sites on the left of a partition, and $N/2$ lattice sites on the right. There are $N/2$ particles, which are indistinguishable from each other, and $N/2$ empty lattice sites.

- (a) Initially, the system is demixed with $N/2$ particles on the right side, and $N/2$ empty sites on the left side. What is the multiplicity of the demixed state?

$$W_d =$$

- (b) Now suppose the particles are free to mix in the total N sites. What is the multiplicity of the mixed state?

$$W_m =$$

- (c) Using your results from (a) and (b), write an expression for the ratio of the multiplicity of the demixed state and the mixed state.

$$\frac{W_d}{W_m} =$$

- (d) Assuming $N \gg 1$, use Stirling's approximation to simplify the result in (c).

$$\frac{W_d}{W_m} =$$

- (e) Using your result from (d), determine the limit of the ratio as $N \rightarrow \infty$.

$$\lim_{N \rightarrow \infty} \frac{W_d}{W_m} =$$

Table 1: Fundamental and derived constants

Name	Symbol	Value	Units
Atomic Mass Unit	u	$1.660\,54 \times 10^{-27}$	kg
Avogadro's Number	N_A	$6.022\,14 \times 10^{23}$	mol ⁻¹
Mass of an Electron	m_e	$9.109\,38 \times 10^{-31}$	kg
Mass of a Neutron	m_n	$1.674\,93 \times 10^{-27}$	kg
Mass of a Proton	m_p	$1.672\,62 \times 10^{-27}$	kg
Faraday's Constant	F	$9.648\,53 \times 10^4$	C/mol
Gas Constant	R	8.314 46	J/(K mol)
		0.082 06	L atm/(K mol)
		62.363 58	L Torr/(K mol)
Boltzmann's Constant	k_B	$1.380\,65 \times 10^{-23}$	J/K
Planck's Constant	h	$6.626\,07 \times 10^{-34}$	J s
Speed of Light	c	$2.997\,92 \times 10^8$	m/s
Earth's Gravitational Constant	g	9.806 65	m/s ²
Bohr Radius	a_0	$5.291\,77 \times 10^{-11}$	m
Rydberg Constant	R_∞	$1.097\,37 \times 10^7$	m ⁻¹
Hartree Energy	E_h	$4.359\,74 \times 10^{-18}$	J

Table 2: Formulas

$\ln 2 = 0.693$	$c_V = \frac{3}{2}R$	$\Delta S_{\text{sys}} = nc_P \ln \left(\frac{T_2}{T_1} \right)$
$\ln 3 = 1.099$	$\Delta U = nc_V \Delta T$	$\Delta S_{\text{sys}} = nc_V \ln \left(\frac{T_2}{T_1} \right)$
$\ln 5 = 1.609$	$\Delta H = nc_P \Delta T$	$\int_0^\infty e^{-\alpha x} dx = \frac{1}{\alpha}$
$\ln N! \approx N \ln N - N$	$w = - \int_{V_1}^{V_2} P_{\text{ext}}(V) dV$	$\int_0^\infty e^{-\alpha x^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{\alpha}}$
$\Delta U = q + w$	$w = -nRT \ln \left(\frac{V_2}{V_1} \right)$	$\int_0^\infty x e^{-\alpha x^2} dx = \frac{1}{2\alpha}$
$H = U + PV$	$w = -P_{\text{ext}} \Delta V$	$\int_0^\infty x^2 e^{-\alpha x^2} dx = \frac{1}{4} \sqrt{\frac{\pi}{\alpha^3}}$
$q = mc_s \Delta T$	$\Delta S_{\text{sys}} = \int_1^2 \frac{dq_{\text{rev}}}{T}$	$N! \approx \left(\frac{N}{e} \right)^N$
$q_{\text{calorimeter}} = C_{\text{calorimeter}} \Delta T$	$\Delta S_{\text{sys}} = \frac{q_{\text{rev}}}{T}$	
$c_P = c_v + R$	$\Delta S_{\text{sys}} = nR \ln \left(\frac{V_2}{V_1} \right)$	
$c_P = \frac{5}{2}R$	$\Delta S_{\text{surr}} = \frac{q_{\text{surr}}}{T_{\text{surr}}}$	$f(x) = \sum_{i=0}^{\infty} \frac{1}{i!} \left. \frac{d^i f(x)}{dx^i} \right _{x=a} (x-a)^i$