Discussion 8

Name and section: ____________________

1. Calculate q_{rev}, ΔU, and ΔS for the reversible cooling of one mole of an ideal gas at constant volume V_1 from (P_1, V_1, T_1) to (P_2, V_1, T_2) followed by a reversible expansion at constant pressure P_2 from (P_2, V_1, T_2) to (P_2, V_2, T_1) (the final state for all the processes are shown in fig. 1). Compare your result for ΔS with that of path A.

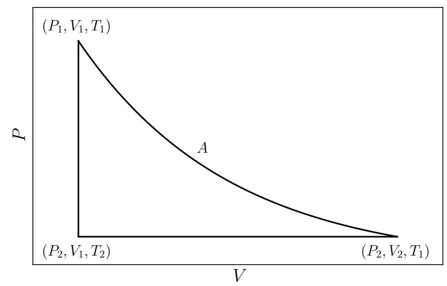


Figure 1: Pressure vs. volume for three processes

Solution: Considering the isochoric process as process 1 and the isobaric process as process 2:

$$
\begin{aligned}
\Delta U_1 &= C_V(T_2 - T_1)\\
\Delta U_2 &= C_V(T_1 - T_2)\\
w_1 &= -\int_1^2 P\,\mathrm{d}V = 0\\
w_2 &= -P_2(V_2 - V_1)\\
q_1 = \Delta U_1 &= C_V(T_2 - T_1)\\
q_2 = \Delta U_2 - w_2 &= C_V(T_1 - T_2) + P_2(V_2 - V_1)\\
&= \frac{C_V}{R} P_2(V_2 - V_1) + P_2(V_2 - V_1)\\
&= P_2(V_2 - V_1)\left(\frac{C_V}{R} + 1\right)\\
\mathrm{d}S_1 &= \frac{C_V}{T}\,\mathrm{d}T\\
\Delta S_1 &= \int_{T_1}^{T_2} \frac{C_V}{T}\,\mathrm{d}T\\
&= C_V \ln\left(\frac{T_2}{T_1}\right)\\
\mathrm{d}S_2 &= \frac{C_V}{T}\,\mathrm{d}T + \frac{P}{T}\,\mathrm{d}V
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{C_V}{T}\,\mathrm{d}T + \frac{R}{V}\,\mathrm{d}V \\
\Delta S_2 &= \int_{T_2}^{T_1} \frac{C_V}{T}\,\mathrm{d}T + \int_{V_1}^{V_2} \frac{R}{V}\,\mathrm{d}V \\
&= C_V \ln\left(\frac{T_1}{T_2}\right) + R\ln\left(\frac{V_2}{V_1}\right) \\
&= C_P \ln\left(\frac{T_1}{T_2}\right)
\end{aligned}
$$

Combining processes 1 and 2:

$$
\begin{aligned}
\Delta U &= 0 \\
w &= -P_2(V_2 - V_1) \\
q &= P_2(V_2 - V_1) \\
\Delta S &= R\ln\left(\frac{V_2}{V_1}\right)
\end{aligned}
$$

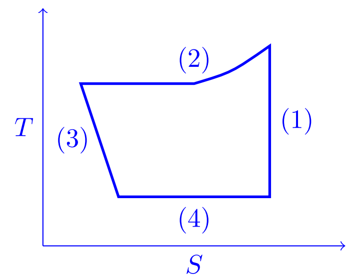
For process A:

$$
\begin{aligned}
\mathrm{d}S &= \frac{1}{T}\,\mathrm{d}T + \frac{P}{T}\,\mathrm{d}V - \frac{\mu}{T}\,\mathrm{d}N \\
&= \frac{P}{T}\,\mathrm{d}V \\
\Delta S &= \int_{V_1}^{V_2} \frac{R}{V}\,\mathrm{d}V \\
&= R\ln\left(\frac{V_2}{V_1}\right)
\end{aligned}
$$

2. (1) A circulating refrigerant such as Freon enters a compressor as a vapor. The vapor is compressed at constant entropy and exits the compressor. (2) The vapor travels through the condenser which first cools and then condenses the vapor into a liquid by removing additional heat at constant pressure and temperature. (3) The liquid refrigerant goes through the expansion valve (also called a throttle valve) where its pressure abruptly decreases, causing flash evaporation adiabatically. That results in a mixture of liquid and vapor at a lower temperature and pressure. (4) At constant pressure, cold liquid-vapor mixture then travels through the evaporator coil or tubes and is completely vaporized by cooling warm air (from the space being refrigerated) blown by a fan across the evaporator coil or tubes. (1) The resulting refrigerant vapor returns to the compressor inlet and is reversibly compressed to complete the thermodynamic cycle.

(a) Draw this cycle on a temperature-entropy diagram.

Solution:



(b) What variables can you define for each part of this cycle?

Table 1: Fundamental and derived constants

Name	Symbol	Value	Units
Atomic Mass Unit	u	$1.660\,54 \times 10^{-27}$	kg
Avogadro's Number	N_A	$6.022\,14 \times 10^{23}$	$\mathrm{mol^{-1}}$
Mass of an Electron	m_e	$9.109\,38 \times 10^{-31}$	kg
Mass of a Neutron	m_n	$1.674\,93 \times 10^{-27}$	kg
Mass of a Proton	m_p	$1.672\,62 \times 10^{-27}$	kg
Faraday's Constant	F	$9.648\,53 \times 10^{4}$	C/mol
Gas Constant	R	$8.314\,46$	J/(K mol)
		$0.082\,06$	L atm/(K mol)
		$62.363\,58$	L Torr/(K mol)
Boltzmann's Constant	k_B	$1.380\,65 \times 10^{-23}$	J/K
Planck's Constant	h	$6.626\,07 \times 10^{-34}$	J s
Speed of Light	c	$2.997\,92 \times 10^{8}$	m/s
Earth's Gravitational Constant	g	$9.806\,65$	$\mathrm{m/s^2}$
Bohr Radius	a_0	$5.291\,77 \times 10^{-11}$	m
Rydberg Constant	R_∞	$1.097\,37 \times 10^{7}$	$\mathrm{m^{-1}}$
Hartree Energy	E_h	$4.359\,74 \times 10^{-18}$	J

Table 2: Formulas

$$\ln 2 = 0.693$$

$$\ln 3 = 1.099$$

$$\ln 5 = 1.609$$

$$\ln N! \approx N \ln N - N$$

$$\Delta U = q + w$$

$$H = U + PV$$

$$q = m c_s \Delta T$$

$$q_{\text{calorimeter}} = C_{\text{calorimeter}} \Delta T$$

$$c_P = c_v + R$$

$$c_P = \frac{5}{2} R$$

$$c_V = \frac{3}{2} R$$

$$\Delta U = n c_V \Delta T$$

$$\Delta H = n c_P \Delta T$$

$$w = -\int_{V_1}^{V_2} P_{\text{ext}}(V)\,\mathrm{d}V$$

$$w = -nRT \ln\left(\frac{V_2}{V_1}\right)$$

$$w = -P_{\text{ext}} \Delta V$$

$$\Delta S_{\text{sys}} = \int_1^2 \frac{\mathrm{d}q_{\text{rev}}}{T}$$

$$\Delta S_{\text{sys}} = \frac{q_{\text{rev}}}{T}$$

$$\Delta S_{\text{sys}} = nR \ln\left(\frac{V_2}{V_1}\right)$$

$$\Delta S_{\text{surr}} = \frac{q_{\text{surr}}}{T_{\text{surr}}}$$

$$\Delta S_{\text{sys}} = n c_P \ln\left(\frac{T_2}{T_1}\right)$$

$$\Delta S_{\text{sys}} = n c_V \ln\left(\frac{T_2}{T_1}\right)$$

$$\int_0^\infty \mathrm{e}^{-\alpha x}\,\mathrm{d}x = \frac{1}{\alpha}$$

$$\int_0^\infty \mathrm{e}^{-\alpha x^2}\,\mathrm{d}x = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}$$

$$\int_0^\infty x\mathrm{e}^{-\alpha x^2}\,\mathrm{d}x = \frac{1}{2\alpha}$$

$$\int_0^\infty x^2\mathrm{e}^{-\alpha x^2}\,\mathrm{d}x = \frac{1}{4}\sqrt{\frac{\pi}{\alpha^3}}$$

$$N! \approx \left(\frac{N}{\mathrm{e}}\right)^N$$

$$f(x) = \sum_{i=0}^{\infty} \frac{1}{i!} \left.\frac{\mathrm{d}^i f(x)}{\mathrm{d}x^i}\right|_{x=a} (x-a)^i$$