1. Find the point \((x^*, y^*, z^*)\) that is at the minimum of the function

\[ f(x, y, z) = 2x^2 + 8y^2 + z^2 \]  

subject to the constraint equation

\[ g(x, y, z) = 6x + 4y + 4z - 72 = 0 \]  

2. A circle is centered about the axes and satisfies the equation

\[ x^2 + y^2 = 4. \]  

Find the point \((x^*, y^*)\) on the circle that is closest to the point \((3, 2)\).
3. You play a slot machine in Las Vegas. For every $1 coin you insert, there are three outcomes:
   1. you lose your $1; net profit of $−1$
   2. you win $1; net profit of $0$
   3. you win $5; net profit of $4$.

Suppose you believe that your average expected profit over many trials is $0. Find the maximum entropy distribution for the probabilities $p_1$, $p_2$, and $p_3$ of observing outcomes (1), (2), and (3) respectively. (Hint: What are the two constraints for the problem?)