Name and section: ____________________

1. In class we discussed how a large number of coin flips will make the distribution much more strongly peaked than a smaller number of coin flips. Is the same true for the sums of dice rolls?

   (a) How many ways can you roll a sum of 13 with three dice? Let's call this $N(3,13)$. Likely the easiest way to do this is to recognize that $N(3,13)$ is the coefficient of x^{13} in the expansion of

   $$(x + x^2 + x^3 + x^4 + x^5 + x^6)^3. \tag{1}$$

   Programs such as Mathematica (available for free to BU students) and websites such as Wolfram Alpha can be very useful for this type of thing.

   (b) Now that we've shown how to find $N(d,s)$, back to the original question. What is the ratio $N(8,28)/N(8,18)$? Note, the sums 28 and 18 are the most likely and halfway between the minimum and the most likely, respectively.

   (c) What is the ratio $N(16,56)/N(16,36)$? Note again that these are the most likely and halfway to the most likely.

2. Consider two identical regions of space initially separated by a removable wall. Suppose the left region is represented by M lattice sites and the right region is represented by an additional M sites. Initially, with the separating wall in place, the left region contains N particles, while the right region contains no particles.

   (a) Write down an expression for the total number of configurations, W, for this system.

   (b) If the entropy, S, of the system is related to W by $S = k_B \ln W$, where k_B is the Boltzmann constant, using Stirling's approximation, calculate the entropy S_i of the initial state described above.

   (c) Now the wall is removed and the system equilibrates to its final state where its entropy is S_f . Calculate S_f.

   (d) Compute the change in entropy ΔS. Simplify ΔS by assuming the ideal gas limit $M \gg N$ so that $1 - N/M \approx 1$

3. Suppose that you have $2V$ black particles and $2V$ white particles in $4V$ lattice sites. There are $2V$ lattice sites on the left and $2V$ lattice sites on the right, separated by a permeable wall. The total volume is fixed. Show that perfect de-mixing (all white on one side, all black on the other) becomes increasingly improbable as V increases. (Hint: refer to Example 2.3 in the book.)

   (a) If it helps, start by thinking about the cases you can easily draw all the possibilities ($V = 1$ and $V = 2$, for example). Which is more likely to be unmixed?

   (b) Now think about the general case of V sites. Show (mathematically) how they are unlikely to demix assuming no interparticle interactions.

   (c) Starting to think about *enthalpic* versus *entropic* contributions to the driving forces, which driving force makes demixing unlikely? How might the other force contribute to demixing?

4. Find the value $n = n^*$ that causes the function

$$W(n) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n} \tag{2}$$

to be a maximum, for constants p and N. Use Stirling's approximation, $x! \approx (x/e)^x$. Note that it is easier to find the value of n that maximizes $\ln W$ than the value that maximizes W. Will the value of n^* will be the same?