A paradox of thrift in general equilibrium without forward markets[1]

Christophe Chamley

Boston University and PSE

February 2, 2011

Abstract

Since 2008, the US personal saving rate had its strongest post-war jump, from 2 to 5 percent, and the investment ratio its sharpest fall from its post-war average of 16 percent to its lowest level of 12 percent. The coordination of saving and investment is analyzed here in a theoretical model of general equilibrium with rational expectations and no forward market. Shocks affect preferences for future consumption. A paradox of thrift is proven that formalizes an argument in the General Theory of Keynes but the equilibrium is a constrained Pareto optimum. Textbook fiscal policies are neutral at best, or inefficient.

Keywords: Paradox of thrift, coordination, incomplete markets, neutral fiscal policy.

[1] This paper would not have appeared without a stimulating discussion with Jacques Drèze. I am very grateful to the refereeing process that was especially productive.

1 Introduction

The gradual decline of the US personal saving rate from 10 percent in the seventies to 2 percent around 2006 has been sharply reversed since 2008 with a jump to more than 5 percent in the last three years. For these three years, the ratio of gross domestic investment to GDP in the US has suddenly collapsed from its post WWII average of 16 percent to 12 percent, the lowest level[2] since 1947. Yet, it is common knowledge (in 2010), that U.S. companies are "sitting on a large pile of cash".

Similar data for other economies have reminded us that the coordination between saving and investment is a central issue in macroeconomics. The fall of consumption and its impact on investment has revived the debate on the paradox of thrift. In commentaries and policy discussions, a recurrent theme is the alleged positive effect of consumption on investment through expectations (*e.g.*, firms have accumulated large amounts of liquid assets without investing, some countries save "too much"). The reduction of private spending and the recession has motivated policies of government spending. These issues are not new. They go back to Mandeville (1714) and Keynes (1936) for whom

[2] All data is from "FRED" of the Federal Reserve Bank of St Louis.

"an act of individual saving means — so to speak — a decision not to have dinner today. But it does *not* necessitate a decision to have dinner or to buy a pair of boots a week hence or a year hence or to consume any specified thing at any specified date. Thus it depresses the business of preparing to-day's dinner without stimulating the business of making ready for some future act of consumption. [...] The expectation of future consumption is so largely based on current experience of present consumption that a reduction in the latter is likely to depress the former, with the result that the act of saving will not merely depress the price of consumption-goods and leave the marginal efficiency of existing capital unaffected, but may actually tend to depress the latter also. In this event it may reduce present investment-demand as well as present consumption-demand. [...] An individual decision to save does not, in actual fact, involve the placing of any specific forward order for consumption, but merely the cancellation of a present order. Thus, since the expectation of consumption is the only *raison d' être of employment*, there should be nothing paradoxical in the conclusion that a diminished propensity to consume has *cet. par.* a depressing effect on employment"[3].

This description which makes no reference to money, interest rate or price rigidity corresponds to some extent to the standard textbook account of Samuelson (1980). In that presentation, the paradox of thrift rests on a positive relation between aggregate investment and income. Such a positive relation is not generated by a structural model however, and without an analysis of market structure or failure, one cannot assess the

[3] Keynes (1936), Chapter 16. The quote is the only mention of a paradox of thrift in the General Theory.

internal consistency of the argument nor analyze the efficiency of the outcome.

The purpose of the present paper is not an exercise in hermeneutics on what Keynes wrote or thought but to analyze the intuitive story of Keynes in a structural model with rational expectations. That analysis will show that the argument may require an additional assumption to be correct, beyond the absence of forward markets, and that the apparent malfunction of an economy may still be a constrained Pareto optimum.

The possibility of a paradox of thrift has been recently analyzed through structural models that are called "new Keynesian" because they assume a slow adjustment of nominal prices (Christiano, 2009, Eggertson, 2009). In these models, the government spending multiplier may be large because of nominal rigidities when the nominal interest rate is constant, either through a policy of the central bank or because it is constrained to be positive. These properties are derived under perfect foresight for agents. The focus of this paper is totally different: prices adjust perfectly, there is no money and the uncertainty with imperfect information due to incomplete markets is key for the properties of the model.

The analysis of uncertainty and irreversible investment on aggregate fluctuations in a structural model has been revived recently. In Bloom (2010), random shock affects the production technology for the aggregate economy and for specific sectors. In the present paper, uncertainty affects the consumption function (as in the paradox of thrift), and the emphasis is on the missing future markets. Paulsen (1989) analyzes the general equilibrium with irreversible investment in production and a money market when the shocks affect the technology of production.

Some studies have interpreted coordination problems within models of multiple equilibria that can be Pareto ranked. Such multiplicity can be generated by increasing returns with or without imperfect competition with complete markets[4]. In the present paper, there is a unique equilibrium, and perfect competition will be assumed in order to highlight the role of missing forward markets[5].

The model is presented in Section 2 and allows for two types of market structures. The first has complete markets for future goods. The second has incomplete markets: there is a market for bonds between today and tomorrow, but, following Keynes, there is no market for the future delivery of specific goods. Goods consumed in future periods are traded in spot markets. This incompleteness is a common feature of actual economies.

In Section 3, the economic mechanisms are analyzed in an intuitive description of the equilibria under the two structures and the preference shocks. The formal analysis is presented in Section 4. Under complete markets, the consumers' plans are conveyed perfectly to investors. Demand shocks increase the level of investment.

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[4]Cooper and John (1989) provide a general framework and survey, under common knowledge. For a switch between regimes without common knowledge, see Chamley (1999).

[5]In Chamley (2011), an increasing of the motive for saving in money is self-fulfilling and generates multiple equilibria.

In the economy with spot markets, information is imperfect when a shock occurs: a shift of consumption preferences toward the future introduces uncertainty on the type of good that is preferred or the timing of consumption. This uncertainty lowers the marginal efficiency of investment, and thus depresses the firms' demand for investment funds. Proposition 1, which is the main result of the paper, shows that under suitable parameter values, the downward shift of the demand for funds dominates the upward shift of the supply. Hence, investment (which is equal to saving), the interest rate and possibly employment, all decrease. The properties of the model thus match the previous description of Keynes.

The option value of delaying irreversible investment by an individual agent or a firm has been analyzed in Bernanke (1983) in a partial equilibrium setup, and in a model of learning from others by Chamley and Gale (1994). In the present paper, there is no individual delay. There is an economy-wide delay in the inputs for production, but this delay operates across agents in general equilibrium.

As shown in Section 5, the paradox of thrift that occurs when consumption preference shift from the present toward the future does not imply inefficiency: the equilibrium is a Pareto optimum when the policy is based on the information that is revealed within the structure of incomplete markets and does not change the information revealed by these markets. Although the economy exhibits "Keynesian" properties, textbook policies such as debt financed public expenditures are not efficient.

2 The model

In the previous description of Keynes, a reduction of aggregate consumption in the present is generated by a shift of consumers' preferences toward the future. The critical element in the story is that in an economy with incomplete markets, producers while observing the drop of consumption do not know the time profile of aggregate preferences for future consumption. In the construction of the canonical model, we incorporate only the features that are essential for the analysis. The incompleteness of markets will be captured by a one period bond market, between consecutive periods, and the uncertainty about the timing of future demand requires at least two periods beyond the present.

Preferences of consumers over consumptions in the three periods, 0, 1 and 2, are affected by a random shock in period 0. In period 0, consumers know their preferences, but a one-period bond market will not be able to convey to producers the relative preferences of agents between the consumptions in the two future periods, 1 and 2. In period 1 however, the one-period bond market (between periods 1 and 2) reveals these preferences to the producers and there is not more uncertainty in that period.

This setting is equivalent to a model with two periods, 0 and 1, where in the future period 1, there are two goods, 1 and 2, for which there is a spot market in period 1 (as the one-period bond market between aggregate consumptions in period 1 and 2). The economy with two periods and two goods in period 1, (goods 1 and 2), is equivalent to an economy with three periods and one consumption good per period with a bond market between consecutive periods. Likewise, the two-period economy with a futures market in period 0 for the two goods 1 and 2 is a complete market economy that is equivalent to a three-period economy, one good per period and complete asset markets in period 0 for the consumptions in all future periods.

We think that the issue of incomplete markets is important both in the context of the timing of aggregate consumption and the shift of preferences for some specific goods in the future. The model is therefore relevant for both contexts. For ease of exposition we will use the two-period interpretation.

Consumers

The separation between consumers who save and entrepreneurs who invest is central in the model. In any equilibrium, the supply of saving by consumers depends on the return on savings and on marginal rate of substitution between the present and the future, which we can define here as the ratio between the expected marginal utility of income in the future and the marginal utility today. When shocks affect preferences for future consumption, they may affect this marginal rate of substitution. The direction of the impact is in general ambiguous and depends on the elasticity of substitution between the present and the future. In this paper, we want to focus on difference between the level of saving (investment) under complete and incomplete markets. We therefore choose a structure of individuals' preferences with a unit elasticity of substitution between the present and the future. In this case, the shocks will have no effect on the expected marginal rate of substitution between the present and the future. There is a continuum of mass one of these consumers with $i \in (0,1)$. Each consumer i has a utility function

$$U^i = (1 - \alpha^i - \beta^i)Log(1 - \ell^i) + \alpha^i Log(c_1^i) + \beta^i Log(c_2^i), \tag{1}$$

where c_k^i is the individual's consumption of good k and ℓ^i is his labor supply in period 0. The time endowment is equal to 1 in period 0 and to T in period 1. One could also assume that consumers are endowed with one unit of good in period 0, consume $1 - \ell^i$ and save ℓ^i. For simplicity and without loss of generality, the individual is assumed to derive no utility from leisure in period 1.

The individual parameters α^i, β^i are such that

$$\alpha^i = \alpha + \epsilon_1^i, \qquad \beta^i = \beta + \epsilon_2^i, \tag{2}$$

where α and β are aggregate parameters, and ϵ_k^i are idiosyncratic random parameters that are independent of α and β.

There are three aggregate states: State 0 is the "normal" state of the economy with no aggregate shock of preferences. In that state, $\alpha = \beta = \gamma_0/2$. In states 1 and 2, a preference shock takes place that affects the aggregate component of the weight of the consumption goods 1 and 2. For simplicity, the shocks are symmetric. In both states $\alpha + \beta = \gamma_1$ where $\gamma_1 \neq \gamma_0$. If $\gamma_1 > \gamma_0$, the shock shifts the taste toward future goods in the aggregate. That is the case of interest here, but the working of the model does not depend on the sign of $\gamma_1 - \gamma_0$.

In state 1, consumers' preferences move on average toward good 1: $\alpha = \gamma_1/2 + \eta$ and $\beta = \gamma_1/2 - \eta$, where η is a fixed parameter. In state 2, $\alpha = \gamma_1/2 - \eta$ and $\beta = \gamma_1 + \eta$. The probabilities of states 1 and 2 are identical, for symmetry. The probability of a preference shock, (*i.e.*, the state different from 0), will not matter because the occurrence of a such shock will be revealed by the market. (Whether the shock increases or decreases the demand for a particular good will not be public information under incomplete markets). The preference parameters are realized before the beginning of period 0.

The variances of the idiosyncratic parameters ϵ_k^i are assumed to be large such that an individual's observation of his preference parameters (α^i, β^i) conveys a negligible amount of information about the realization (α, β). To simplify, we will assume the limit case that no consumer, and *a fortiori* no entrepreneur has private information about the realization of the aggregate parameters (α, β).

Entrepreneurs

The other key element in the model is that the consumption goods are produced by entrepreneurs who have access to two technologies, the long-term technology that requires investment in period 0 for production in period 1, and the short-term technology that takes place in period 1 only. Without loss of generality, one can assume that there are two representative price-taking entrepreneurs, one for the long-term and one for the short-term technology. The long-term process is technologically more efficient than the short-term technology, in a sense to be specified below.

In the long-term technology, the production of the quantity y_k of good k in period 1 requires an investment in period 0 that is the quantity of labor ℓ_k with

$$\ell_k = y_k^{1+a}, \qquad \text{with} \qquad a > 0. \tag{3}$$

The short-term technology is linear, for simplicity: the production y'_k of good k in period 1 require the amount of labor z_k with

$$z_k = By'_k, \tag{4}$$

where the marginal cost B will be specified later. The labor market in period 1 is perfectly competitive. Both entrepreneurs are risk-neutral and use their profit to consume a good in period 1 that is produced one-for-one by the labor endowment of consumers in period 1. (The profit of the short-term technology will be nil in equilibrium because of the linearity of production). We also assume that entrepreneurs (or the representative entrepreneur in period 0), have a sufficient endowment in period 1 to cover possible losses: there is no bankruptcy.

The total supply of good k in period 1 is the sum of the productions by the long-term and the short-term technologies:

$$c_k = y_k + y'_k. \tag{5}$$

Assets and Markets

Equilibria will be analyzed for two different market structures. In both cases, markets open after the realization of the state that determines the aggregate consumption parameters α and β. In the first structure, markets are complete. As in Arrow (1964), the completeness is achieved by an asset to carry wealth between the two periods, called a bond, and forward markets, in period 0, between this asset and the two goods produced in period 1. The equilibrium of markets in period 0 will reveal perfectly the state of nature.

In the second structure, one makes the realistic assumption that there is no forward market in period 0 for the consumption goods in period 1. There are only spot markets, the spot market in period 0 for the bond, and the spot markets in period 1 for the two consumption goods. The absence of forward markets is a common feature of an actual economy. In a theoretical model, it can be justified by locational constraints when consumers buy future consumption goods in locations that are different from the ones where they are working. The bond in the present model can fulfill the function of money that is earned on some market and spent on another one. The present model could be extended to include such a segmentation as in Townsend (1980)[6].

Both market structures include the bond to carry wealth from period 0 to period 1. That bond can be defined as a delivery to a basket of goods in period 1. In the present model, it is natural and simplifies the algebra to define the bond as the claim to one unit of labor in period 1, or the good that is produced one for one with labor and consumed by entrepreneurs. The price of the bond in terms of period 0 labor is q. The interest rate is defined by $r = 1/q - 1$.

3 Equilibria: an intuitive description

The equilibrium between saving and investment is obtained by the bond price. We consider first the supply of saving and then the demand for investment.

[6] The interaction between individual uncertainty and aggregate demand is analyzed in Chamley (2010) within a Townsend type model.

Taking the labor in period 1 as numéraire, let p_1 and p_2 be the prices of the two consumption goods, either in forward markets (under complete markets) or in spot markets (when there are no forward markets). A consumer i who saves a quantity of bonds b^i and is endowed with an amount of labor T in period 1, has an endowment $T + b^i$ in period 1. His consumptions of the two goods are

$$c_1^i = \frac{\alpha^i}{\alpha^i + \beta^i}\frac{T + b^i}{p_1}, \qquad c_2^i = \frac{\beta^i}{\alpha^i + \beta^i}\frac{T + b^i}{p_2}. \tag{6}$$

By substitution in the utility function (1), his welfare in period 0 is of the form

$$V^i = (1 - \alpha^i - \beta^i)Log(1 - qb^i) + (\alpha^i + \beta^i)Log(b^i + T) + E[Q^i(p_1, p_2)], \tag{7}$$

where $Q^i(p_1, p_2)$ depends only on the prices and the expectation term is replaced by certainty in the economy with forward markets. The key property of this function, as discussed in the construction of the model, is the separability between the income and the price terms in period 1. It follows immediately that the saving $s^i = qb^i$ is a function of the bond price q:

$$s^i(q) = \alpha^i + \beta^i - (1 - \alpha^i - \beta^i)Tq. \tag{8}$$

Substituting in the demand functions (6), with $b^i = s^i/q$,

$$c_1^i p_1 = \alpha^i(T + \frac{1}{q}), \qquad c_2^i p_2 = \beta^i(T + \frac{1}{q}). \tag{9}$$

By integration over all consumers $i \in (0, 1)$, the idiosyncratic terms in (2) cancel out, and we have the following lemma.

Lemma 1 (supply of saving)

The level of aggregate saving in the economy with forward markets and in the economy with spot markets is a function of the bond price q*:*

$$s(q) = \gamma - (1-\gamma)Tq, \quad \text{with } \gamma = \alpha + \beta. \tag{10}$$

The demand functions in period 1 *satisfy the relations*

$$c_1 p_1 = \alpha(T + \frac{1}{q}), \qquad c_2 p_2 = \beta(T + \frac{1}{q}), \tag{11}$$

where (p_1, p_2) *are the forward prices or the spot prices in terms of period* 1 *labor.*

The saving function is a linear decreasing in q, that is increasing in the interest rate r. It is represented in Figure 1.

Investment has to be made in period 0 for the production of each specific good in period 1. Given the technology specification (3), the profit from investment toward producing a quantity y_k in period 1 is $E[p_k]y_k - (1+r)y_k^{1+a}$ with $1 + r = 1/q$ if there are no forward markets. If there are forward markets, the expected value $E[p_k]$ of the price of good k in the spot market is replaced by the forward market price p_k. The demand for investment toward to the production of good k is $\ell_k = y_k^{1+a}$ and satisfies the equation

$$\ell_k = y_k E[p_k] \frac{q}{1+a}. \tag{12}$$

Total investment is the sum of the investments toward the production of each good.

Lemma 2 (demand for investment)

In the economy without forward markets, the total demand for investment satisfies the equation

$$\ell = (y_1 E[p_1] + y_2 E[p_2]) \frac{q}{1+a}, \tag{13}$$

where p_i are the spot prices of the two goods. In the economy with forward markets, $E[p_i]$ is replaced by the forward price p_i.

Total investment in the long-term technology is linear in the expected value of the output produced by that technology, and it increases linearly with the price of bonds q (which is the inverse of the return $1 + r$), as represented in Figure 1. Using Lemmata 1 and 2, one can describe intuitively the main result of the paper.

A preference shock shifts the supply of saving upwards: the parameter γ increases in the function $s(q)$ that is defined in Lemma 1 and represented in Figure 1. The main argument is about the investment schedule.

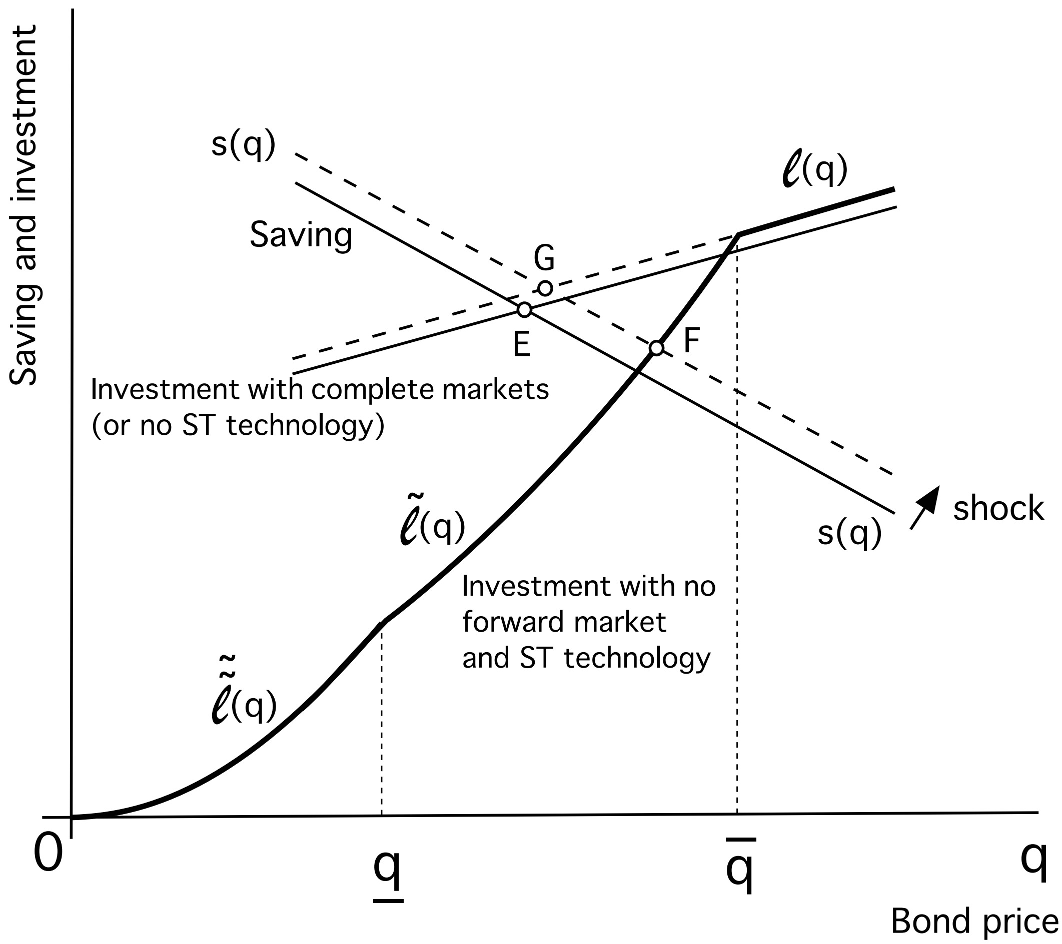
a. Assume first that the long-term technology is the only one that can be used and that there are forward markets. The forward prices reveal perfectly in period 0 the future demand. There is no uncertainty and the demand for investment in Lemma 2 becomes

$$\ell = (y_1 p_1 + y_2 p_2) \frac{q}{1+a}, \tag{14}$$

A preference shock (*i.e.,* the aggregate state 1 or 2), has two components. First, it shifts the demand to the future and thus increases the value of the total demand $y_1p_1 + y_2p_2$. That effect which will be called the *global effect*, increases the demand for total investment. The second effect increases the value of the demand for one of the good and reduces the demand for the other good. It will be called the *sectoral effect*. Given the definition of preferences, an increase of the demand for one good is balanced by the lower demand for the other good and the impact of the sectoral effect on the total value of sales, and therefore on total investment, is nil. However, the sectoral effect has an impact on the composition of investment.

The impact of a shock is to shift upwards the demand for total investment that is defined in Lemma 2. The shifts are represented in Figure 1. The equilibrium moves from the point E to the point G. The level of investment increases. Since the shock is driven by a stronger taste for future consumption, the equilibrium interest rate falls (the bond price rises), as shown in Lemma 3 below.

b. Suppose now that the forward markets are replaced by spot markets while the short-term technology is still at rest. The demand schedule is now determined by (13) and depends on the expected spot prices. When a shock takes place, the bond price reveals that demand has shifted to the future; it reveals the value of γ and the global effect. But the bond price does not reveal the sectoral effect and the direction of the relative changes. An entrepreneur who invests in period 0 faces uncertainty on the value of the demand for his product. Since he is risk-neutral, his investment depends only on the expected value of the sales for which positive and negative sectoral shocks cancel each other out. The total level of investment is the same as in the case (a), but the composition of investment is different: now investment for the two goods are the same.



At the equilibrium E, there is no preference shock. A preference shock with higher taste for total consumption the future (as described in the text) shifts the equilibrium to G under complete markets, and to F under spot markets, with a level of saving lower than at E.

Figure 1: Equilibria with and without forward markets

Because firms cannot prepare the production for a higher of a lower demand in a specific market, the equilibrium adjustment has to be done by the spot prices p_1 and p_2. When supply does not respond to the sectoral shocks but only to the global shock, the variations of the prices must be greater than when the supply can respond to the two types of shocks (in the complete markets economy).

c. We can now bring in the short-term technology. Under complete markets, the prices p_1 and p_2 move in a relatively narrow band. By assumption, the cost of production B of the short-term technology is higher than the maximum of the forward prices p_1 and p_2, and the short-term technology does not operate in equilibrium. When forward markets are replaced by spot markets, the variation of the spot prices p_1 and p_2 is larger and their maximum is greater than the cost of production, B. (If not, the case is trivial). In equilibrium, the short-term technology produces some of the good in high demand. That technology puts effectively a cap on the maximum of the spot prices. That cap has a negative impact on the expected value of any spot price and therefore a negative impact on the demand for investment. The effect is illustrated in Figure 1 where the downward shift of the investment schedule dominates the upward shift of the supply of saving: the shock moves the economy from the point E to the point F; total investment falls and the bond price increases.

Figure 1 thus illustrates the paradox of thrift that was introduced at the beginning of the paper: a preference shock towards future consumption increases the level of investment in the complete markets economy and lowers investment in the spot markets economy. In both cases, the bond price rises. (The interest rate falls). These mechanisms will be discussed again in the concluding section. The next section provides the analytics of the intuitive description.

4 Analysis

4.1 The economy with forward markets

Assume first that the short-term technology (production in period 1) is shut out because of a high production cost. (A sufficient condition will be given later). The production y_k is equal to the consumption c_k which satisfies the equations in (11) that are known by entrepreneurs. Using these expressions with (14), the total demand for investment in period 0 can be expressed as a function of the bond price q.

$$\ell(q) = \frac{\gamma}{1+a}(1+qT). \tag{15}$$

The demand for investment is a linear function of the bond price q, as represented in Figure 1. The equilibrium bond price is determined by the equality between the demand $\ell(q)$ and the supply $s(q)$ in Lemma 1. Given the price q, one can compute the values of the consumptions $c_k p_k$ in Lemma 1. Since $c_k = y_k$, using the investment equation (12) and straightforward algebra, one finds the values of ℓ_k and therefore the outputs $y_k = c_k$ and the prices p_k. The results are presented in the next Lemma.

Lemma 3

In an economy with forward markets and with no short-term technology, the equilibrium bond price is equal to

$$q^* = \frac{a\gamma}{T(1+a(1-\gamma))}.$$

- *If there is no shock, ($\alpha = \beta = \gamma_0/2$), then*

$$p_1 = p_2 = p(\gamma_0), \quad \textit{with} \quad p(\gamma) = \Big(\frac{1+a(1-\gamma)}{\gamma}\Big)^{1/(1+a)} \Big(\frac{1}{2}\Big)^{a/(1+a)} \frac{(1+a)T}{a}. \tag{16}$$

- *If there is a shock ($\alpha = \gamma_1/2 \pm \eta,\ \beta = \gamma_1 - \alpha$), the prices of the good in high and low demand are*

$$p_h = \Big(1 + \frac{2\eta}{\gamma_1}\Big)^{a/(1+a)} p(\gamma_1), \qquad p_\ell = \Big(1 - \frac{2\eta}{\gamma_1}\Big)^{a/(1+a)} p(\gamma_1), \tag{17}$$

  where the function $p(\cdot)$ is given in (16).

We assume that the price of good in high demand when there is a shock is higher than the price when there is no shock. The following Lemma provides a condition.

Lemma 4

The price of the good in high demand increases when there is a shock, $p_h > p(\gamma_0)$, if and only if

$$\gamma_1 - \gamma_0 < 2\frac{\eta}{\gamma_1}\frac{1+a(1-\gamma_1)}{1+a}. \tag{18}$$

The condition (18) which is not very restrictive[7], is assumed to hold.

The short-term technology is not used if its cost of production is higher than the price of the good in high demand. To focus on the main argument of the paper, we posit that the cost of production in the short-term technology, B, is not sufficiently low for the operation of that technology when forward markets eliminate any uncertainty on future demand. We have the following result.

Lemma 5

If the cost of the short-term technology B is higher than p_h defined in (22), then in an equilibrium with forward markets, the short-term technology does not operate and the equilibrium prices are given in Lemma 3.

4.2 The economy without forward markets

As in the previous section, consider first the case where the short-term technology (production in period 1) is shut out because of a high production cost. The equation (15) of the demand for total investment is now replaced by

$$\ell(q) = \frac{E[\gamma]}{1+a}(1+qT), \tag{19}$$

[7]The condition holds for any a if $\gamma_1 < (1+\gamma_0)/2$. The assumption (18) simplifies the presentation, but is not necessary for the main property of the model.

and the equilibrium equation between $s(q)$ in (10) and the demand for investment $\ell(q)$ in (15) takes the form

$$\gamma - (1-\gamma)T = \frac{E[\gamma]}{1+a}(1+qT). \tag{20}$$

In a rational expectation equilibrium, this equation with the observation of the price q reveals the true value of $\gamma \in \{\gamma_0, \gamma_1\}$, that is whether a shock has occurred. The property is generic for any utility function that produces a saving function which depends on the "average preference" for the future and on the one period interest rate. The parameter γ represents that average preference and the entrepreneur's demand for investment depends on their expectation of this average. Through the market equilibrium, entrepreneurs who know the structure of the equilibrium can compute exactly the parameter γ of the average preferences of the consumers. Therefore, we can replace equation (20) by

$$\gamma - (1-\gamma)T = \frac{\gamma}{1+a}(1+qT). \tag{21}$$

The bond market equilibrium is represented in Figure 1 by the same aggregate demand and supply schedules as in the case of complete markets and the equilibrium bond price is the same as in the setting with complete market. The composition of the investment is different however because without forward markets, the bond price does not reveal which good is favored by the shock[8].

[8]That property is robust in a more general model where α and β are two random variables. The two-dimensional uncertainty has been simplified to a symmetric shocks on α and β to the sake of exposition.

Entrepreneurs use the demand functions (11) to determine the expected price of their period 1 output. When there is a shock, conditional on the observation of the bond market period 0, $E[\alpha] = E[\beta] = \gamma_1$. Using a simple algebra similar to that of Lemma 3, the equilibrium is characterized by the next result.

Lemma 6

In the economy without forward markets, if the short-run technology does not operate, the equilibrium bond price is the same as in the economy with forward markets, and the period 1 *spot prices of the consumption goods in high and low demand, respectively, are given by*

$$\hat{p}_h = \Big(1 + \frac{\eta}{2\gamma_1}\Big)p(\gamma_1), \qquad \hat{p}_\ell = \Big(1 - \frac{\eta}{2\gamma_1}\Big)p(\gamma_1), \tag{22}$$

where the function $p(\cdot)$ is given in (16) of Lemma 3.

Comparing these expressions with those in the economy with forward markets (17), because the marginal cost of investment is an increasing function ($a > 0$), we have

$$\hat{p}_\ell < p_\ell \quad \text{and} \quad p_h < \hat{p}_h.$$

The shocks have a larger impact on the spot prices than on the forward prices: producers cannot anticipate perfectly the demand shocks for particular goods as in the complete markets economy and larger price variations are required to absorb the shocks.

The operation of the short-term technology depends on its production cost, B. If B is higher than the highest price $\hat{p}_h$, that technology never operates. The interesting case is the intermediate value of B, between p_h and $\hat{p}_h$. (The properties of the model hold at least for some values $B < p_h$, but the exposition becomes somewhat technical in that case). Given the values of p_h in (17) and $\hat{p}_h$ in (22), we make the following assumption.

Assumption 1 *$p_h < B < \hat{p}_h$, with p_h and $\hat{p}_h$ defined in Lemma 3 and Lemma 6.*

In the absence of forward markets, entrepreneurs are aware of a shock, but in the absence of further information, investment for each good is half the total investment. The absence of forward markets has no impact on the supply of saving, but it has a significant impact on the demand for investment which we now analyze.

- If q is sufficiently high, which is equivalent to a low interest rate, the production scale of the long-term technology is sufficiently high such that the price of the good in high demand is smaller than the short-term production cost B. With no short-term production, the investment function $\ell(q)$ is the same as in (15) and is a linear function of q. That case is relevant for an interval $[\bar{q}, \infty]$ where $\bar{q}$ is determined in the Appendix.
- When $q < \bar{q}$, the price of the good in high demand is higher than B if there is no short-term production. Therefore, the short-term technology operates and the high price is equal to B. There is an interval $q \in (\underline{q}, \bar{q})$ such that the short-term technology produces a fraction of the good in high demand and puts a cap B on its price. The price cap lowers the rate of return on investment which is an increasing function $\tilde{\ell}(q) < \ell(q)$. (See the Appendix).

- For $q < \underline{q}$, the interest rate is so high and the output by the long-term investment so low that that the short-term technology supplements the long-term technology for both goods. The price cap B on the lower price has a stronger negative impact on the investment schedule which is now $\tilde{\tilde{\ell}}(q) < \tilde{\ell}(q)$.

Under Assumption 1, $\underline{q} < q_G < \bar{q}$ where q_G is the bond price at the point G, and the investment schedule that is relevant for q_G is $\tilde{\ell}(q)$. If there is a shock, saving is equal to investment at the point F. If $\gamma_1 - \gamma_0$ is sufficiently small compared to the uncertainty effect η, the positive effect of a higher saving is dominated by the uncertainty effect. The level of investment falls. The figure illustrates a paradox of thrift: the higher preference for future consumption induces a decrease of saving, in equilibrium.

Proposition 1

In the economy with spot markets, under Assumption 1*, for given* η*, there exists* ζ *such that if* $\gamma_0 < \gamma_1 < \gamma_0 + \zeta$*, a shock from* γ_0 *to* γ_1 *that increases the total preference for future consumption induces, in general equilibrium, an increase of saving in the economy with complete markets, and a reduction of saving in the economy with spot markets.*

The allocation of resources by the market thus induces a reduction of investment when there is uncertainty about future demand. This process is related to the option value of delay for firms that make irreversible investment decisions over the business-cycle (Bernanke, 1983). Note that in the present model, individual agents do not delay. An economy-wide delay in allocation of resources operates through the general equilibrium. This general equilibrium reallocation is the same as the one that would be made by a social planner who delays the commitment of resources for production. That equivalence will generate the result of constrained optimality in the next section.

When fluctuations are driven by changes of the consumer preferences, these changes have opposite effects on the levels of aggregate investment and output in the two economies with and without forward markets, respectively. Saving and investment move pro-cyclically in both economies. However, the interest rate moves counter cyclically in the economy with complete markets and pro-cyclically in the economy with spots markets.

5 Efficiency

An important issue is whether the equilibrium is a constrained Pareto optimum, namely whether there are policies which do not require more information than that provided by the market, and which improve the allocation of resources. Because of the heterogeneity of consumers, we measure the welfare of consumers by their average utility. The criterion is equivalent to the expected utility of an individual before the realization of his taste at the beginning of period 0. The number of agents can then be reduced to two, a consumer with utility function

$$U = (1 - \alpha - \beta)Log(1 - \ell) + \alpha Log(c_1) + \beta Log(c_2),$$

and all entrepreneurs that can be aggregated into a single individual who maximizes profits (measured in labor of period 1).

Proposition 2

In the economy with no forward market and a spot technology that satisfies Assumption 1, the equilibrium is a Pareto optimum under the constraint of the information revealed by that equilibrium: the allocation of resources maximizes the average level of utility over all consumers subject to the information of the market and the utility of entrepreneurs in period 0.

The proof is parallel to that of the first fundamental theorem of welfare. The only minor difference is that the planner's allocation is subject to learning the relative preferences of the two goods c_1 and c_2 in period 1 only. The proof can be done by the reader, but for the sake of completeness, it is provided in the Appendix.

The result implies that macroeconomic policies which do not generate a gain of information cannot improve the allocation of resources in the economy with spot markets. For example, a standard "Keynesian" policy in states of "insufficient demand" is a lump-sum tax subsidy in the first period financed by bond issues. The bonds are repaid by future taxes. This policy is neutral here: it has no impact on the allocation of resources because it has no effect on the informational content of the equilibrium, and on the budget constraint of consumers[9]. Some policies could stabilize fluctuations

[9]Since a downturn of output is associated with a fall of the interest rate, it is possible, but not certain, that the second-best criterion with distortionary taxation would imply a lower tax rate and a deficit in a state of low demand.

in the economy, but stabilization is not a policy goal *per se*.

A central feature of the model and of actual economies is that the number of markets is much smaller than the number of goods (indexed by time). In a generic way, the relatively small number of markets cannot reveal complete information about individual preferences. Geanakoplos and Polemarchakis (1986) have shown that allocations are generally constrained inefficient in a setup with missing markets. One could construct an extension of the present model in which a government policy that does not have superior information alters the information properties of the bond market and generates a Pareto improvement. Suppose that consumers are identical and know the state of nature (but cannot communicate reliably with producers). The government can implement an ad valorem taxation of saving with lump-sum refund that is implemented in period 1 contingent on a preference shock that increases the demand for good 1 in period 1. That policy does not require superior information for the government and generates a welfare cost of price distortion that can be made arbitrarily small with a small the tax rate.

In this policy environment, the supply of saving in period 0 depends on whether the shock increases or decreases the demand for good 1. Hence there are three different equilibrium prices for the three states of nature and the bond price reveals the state in period 0. The equilibrium can be shown to Pareto dominate the equilibrium with ignorance on the type of the shock. By continuity, the property is robust to some distribution of shocks with a continuous density. Such an extension may however not be entirely convincing for a policy application and illustrates that it is not sufficient for policy applications to exhibit an equilibrium that is not Pareto-constrained. The analysis of Pareto improving policies for the coordination of saving and investment in plausible models of incomplete markets remains an agenda for future work.

6 Conclusion

The model presented here shows how an increase in the taste for future consumption can induce opposite responses of investment or saving in economies with and without forward markets, respectively, and how aggregate fluctuations can be amplified by the absence of forward markets. The structure of the model was reduced to the elements that are essential for the analysis of theoretical issues. The model is obviously not suitable for empirical investigations. However, the basic mechanism is general and could be included in a more complex model that would focus on empirical issues.

Although the model has Keynesian properties, textbook prescriptions of government expenditures financed through deficits are not effective. Since the market failure is identified as the incomplete structure of forward markets which leads to imperfect information, policy cannot be effective if it does not alter the informational content of equilibrium prices.

Several issues remain unresolved. First, the analysis policy effectiveness and its applicability to a realistic context remains to be investigated. An especially interesting topic would be to see whether monetary policy could affect the informational content of the menu of assets in a practically useful manner. This approach may eventually validate the approach to monetary policy that is based solely on a portfolio foundation[10].

Second, a better microeconomic underpinning of the absence of forward contracts would be useful. This paper has emphasized that the effectiveness of fiscal policy does not require an informational advantage. However, the policy maker has the power to raise taxes and thus to enforce some contracts between individuals which are not available to private agents. This superiority of a central authority is found also in monetary policy, and has been an important justification for an active policy in models where agents are separated by constraints of time and location such as in Townsend (1980).

Finally, this paper has presented one type of market failure. There may be others which are based on entirely different mechanisms. For example, a reduction of current demand (for future consumption), reduces the cash-flow of firms and has a negative impact on the collateral of investors thus reducing the level of investment. The latter effect (induced by a technology shock), has been analyzed by Bernanke and Gertler [1989]. The development of structural models which generate a property such as the paradox of thrift is of interest *per se* because some of the argument for policy seem to rest on the view that a higher demand (for consumption goods), stimulates expectation and investment. The analysis of restrictions to private contracts which lead to market failures will be required in order to establish a sound basis for the role of macroeconomic policy.

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[10] The approach fails when open-market operations does not change the menu of assets, (Chamley and Polemarchakis [1981]).

APPENDIX

Lemma 3

The equality between the supply of saving and the demand for investment in (10) and (19) implies that the bond price satisfies the equation

$$qT = \frac{\alpha + \beta}{\frac{1+a}{a} - (\alpha + \beta)}, \tag{23}$$

and the levels of investment are equal to

$$\ell_1 = \frac{\alpha}{1 + a(1 - \alpha - \beta)}, \quad \ell_2 = \frac{\beta}{1 + a(1 - \alpha - \beta)}, \quad \ell = \ell_1 + \ell_2. \tag{24}$$

From the demand equation (11), the price of good 1 is

$$p_1 = \frac{\alpha}{c_1}(\frac{1}{q} + T). \tag{25}$$

Since $\ell_1 = c_1^{1+a}$, using the two previous equations,

$$p_1 = \Big(\frac{\alpha}{a}\Big)^{a/(1+a)} \frac{(1+a)T}{\alpha + \beta} \Big(\frac{1+a}{a} - (\alpha + \beta)\Big)^{1/(1+a)}.$$

The demand for investment without forward markets

Since the cost B puts a ceiling on the price of a consumption good, if a shock takes place, the expected price of that good is

$$E[p] = \frac{1}{2}\Bigg(Min\Big(B, \frac{\frac{\gamma_1}{2} + \eta}{y}(\frac{1}{q} + T)\Big) + Min\Big(B, \frac{\frac{\gamma_1}{2} - \eta}{y}(\frac{1}{q} + T)\Big)\Bigg), \tag{26}$$

where y is the production of the good through the long-term technology. (That production is the same for the two goods as determined by $\ell(q)/2$ with $\ell(q)$ given in (15)). In equilibrium, the marginal cost in period 0, $(1+a)y^a$, is equal to the discounted value of a the price, $qE[p]$, and

$$\frac{1+a}{q}y^a = \frac{1}{2}\left(Min\Big(B, \frac{\frac{\gamma_1}{2}+\eta}{y}(\frac{1}{q}+T)\Big) + Min\Big(B, \frac{\frac{\gamma_1}{2}-\eta}{y}(\frac{1}{q}+T)\Big)\right). \tag{27}$$

Since the total investment ℓ (for both consumption goods) is equal to $2y^{1+a}$,

$$\ell = \frac{1}{1+a}\left(Min\Big(Bq(\frac{\ell}{2})^{1/(1+a)}, (\frac{\gamma_1}{2}+\eta)(1+qT)\Big)+Min\Big(Bq(\frac{\ell}{2})^{1/(1+a)}, (\frac{\gamma_1}{2}-\eta)(1+qT)\Big)\right). \tag{28}$$

The equation defines a function $\tilde{\ell}(q)$ that is increasing in q and varies from 0 to ∞ when q varies from 0 to ∞. (Investment is inversely related to the interest rate).

Recall that the function $\ell(q)$ under complete markets or with spot markets and no short-term technology as given in (15), satisfies the relation.

$$\ell = \frac{1}{1+a}\Big((\frac{\gamma_1}{2}+\eta)(1+qT) + (\frac{\gamma_1}{2}-\eta)(1+qT)\Big).$$

It follows that $\tilde{\ell}(q) \leq \ell(q)$. $\bar{q}$ is characterized by $\tilde{\ell}(\bar{q}) = \ell(\bar{q})$ which is equivalent to

$$B = \frac{\frac{\gamma_1}{2}+\eta}{(\frac{\tilde{\ell}(\bar{q})}{2})^{1/(1+a)}}(\frac{1}{\bar{q}}+T). \tag{29}$$

When the constraint B is binding for both prices, $q < \underline{q}$ which is defined by

$$B = \frac{\frac{\gamma_1}{2}-\eta}{(\frac{\tilde{\ell}(\underline{q})}{2})^{1/(1+a)}}(\frac{1}{\underline{q}}+T), \tag{30}$$

and the investment function is equal to $\tilde{\tilde{\ell}}(q) < \tilde{\ell}(q)$ such that

$$\tilde{\tilde{\ell}}(q) = 2\left(\frac{qB}{1+a}\right)^{\frac{1+a}{a}}.$$

Proposition 2

In the equilibrium, when a shock takes place, the consumer saves ℓ_0 and consumes c_h and c_ℓ of the goods with the stronger and weaker tastes, respectively. Let p_h and p_ℓ the prices of these goods and consider a change of allocation by a social planner that improves strictly the welfare of the consumer conditional on a shock before he knows his individual (α^i, β^i). Let the change of allocation be $(\Delta\ell_0, \Delta c_h, \Delta c_\ell, \Delta y_1, \Delta y_2)$, where Δy_1 and Δy_2 are the changes of outputs in goods 1 and 2 for the long-term technology. Given the increasing marginal cost in the long-term technology, we can assume that $\Delta y_1 = \Delta y_2 = \Delta y$.

Conditional of a shock, by revealed preferences,

$$p_h \Delta c_h + p_\ell \Delta c_\ell > \frac{\Delta \ell_0}{q}.$$

Since the marginal cost in the long-run technology is increasing,

$$\frac{\Delta \ell_0}{q} \geq \frac{2(1+a)c_\ell^a}{q}\Delta y = (B + p_\ell)\Delta y.$$

Therefore,

$$p_h \Delta c_h + p_\ell \Delta c_\ell > (B + p_\ell)\Delta y.$$

Using $p_h = B$,

$$B(\Delta c_h - \Delta y) + p_\ell(\Delta c_\ell - \Delta y) > 0.$$

Since output in period 1 is the sum of the outputs in the long-term and the short-term technologies, $c_k = y_k + y'_k$ in (5),

$$B\Delta y'_h + p_\ell \Delta y'_\ell > 0,$$

with $\Delta y'_h \geq 0$ and $\Delta y'_\ell \geq 0$. (The short-term technology cannot operate on a negative scale, using goods to produce labor). The previous inequality implies that the short-term technology requires a strictly higher amount of labor that must be taken from production of the good consumed by entrepreneurs. Their utility is strictly smaller.

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