CHRISTOPHE CHAMLEY
CÉLINE ROCHON

From Search to Match: When Loan Contracts Are Too Long

A model of lending is presented where loans are established in matches between banks (lenders) and entrepreneurs (borrowers) who meet in a search process. Projects turn out randomly a quick payoff or a long-term payoff that requires a rollover of the loan. The model generates, under proper parameter conditions, two steady states without or with rollover, and rollover is socially inefficient. Under imperfect information, the standard debt contract is privately efficient. However, it extends the domains of equilibria with socially inefficient rollover. The global dynamics displays a continuum of equilibrium paths that each exhibits sudden discontinuities—crises—in which the mass of outstanding loans is reduced by a quantum amount of terminations. Crises have a cleansing effect.

JEL codes: D82, D83, G21
Keywords: search, debt contract, asymmetric information, debt overhang, strategic complementarity, multiple equilibria.

RECENT CRISES OF THE financial sector (e.g., Japan, U.S., others), exhibit the accumulation of low performing loans. Price competition in late 1980s Japan (essentially from a relaxation of interest rate controls and capital market deregulation) led to compressed interest rate margins, and institutions to expand into riskier lending. Lending standards fell when real estate prices surged. When the bubble burst in early 1990s, the banking sector was severely impacted. Instead of calling back nonperforming loans, banks restructured nonviable loans by reducing interest rates and extending maturities. They granted new loans to allow borrowers to repay

We would like to thank seminar participants at the Cleveland Fed, the Max Planck Institute for Research on Collective Goods in Bonn, FEMES 2009 in Tokyo, EEA-ESEM 2009 in Barcelona, ESWC 2010 in Shanghai, the Said Business School, and the Institute for Advanced Studies in Vienna. A. Rampini made suggestions that were critical (in both senses) at the various stages of writing.

CHRISTOPHE CHAMLEY *is at Department of Economics, Boston University, 270 Bay State Road, Boston MA 02215* (*E-mail:* Chamley@bu.edu) *and directeur d'études, EHESS at the Paris School of Economics*. CÉLINE ROCHON *is at Saïd Business School, University of Oxford, and the International Monetary Fund*.

Received December 29, 2009; and accepted in revised form August 10, 2010.

Journal of Money, Credit and Banking, Supplement to Vol. 43, No. 7 (October 2011)
© 2011 The Ohio State University

overdue loans. Weak loan classification rules allowed banks to classify restructured nonperforming loans as performing as soon as the new loans were made.[1]

Nonperforming loans reduce the capacity of financial institutions for lending to new projects that could stimulate growth and help bring the economy out of a protracted state of stagnation. We analyze this issue in a model with search between lenders (banks) and borrowers (entrepreneurs) and imperfect information on the part of lenders.

Gale and Hellwig (1985) show, following Townsend (1979), that in a one-period model where it is costly for a lender to get information about the ability of the borrower to pay, the optimal loan contract is the standard debt contract, with monitoring (bankruptcy) when the entrepreneur does not pay a fixed amount that is set in the contract.[2] The debt contract shifts the payoff of the lender toward the lower part of the distribution of the investment's return, in which the costly verification (bankruptcy) takes place. In order to minimize cost, all the verified return goes to the lender. A key property for our analysis is that this verified return occurs on average with some delay and requires the continuation of lending. The debt contract that saves on verification cost induces the lender to continue lending for projects that are privately profitable to him (according to the payment structure of the debt contract) but are not socially efficient.

The model that is presented here embodies a stylized representation of the previous property with continuous time and an infinite horizon: investment projects yield randomly a payoff that is either fast and relatively high on average or subject to protracted delays and relatively low on average. The undertaking of an investment project requires a match between a bank and an entrepreneur, and such a match is the result of a search from both sides. The matching is achieved through a random process as in Diamond (1982)[3] and Mortensen-Pissarides (1994).

Whether to roll over or to call the loan back and terminate the investment project depends on the opportunity cost of funds. If there are many opportunities for the use of funds in new projects, it may be better (in a privately efficient contract between lender and borrower) to cut short projects that do not provide an early payoff. However, the opportunities for new projects may be lower when a large mass of borrowers (entrepreneurs) and lenders (banks) are tied in a relation with loan rollover. Hence, the loan rollover may be privately efficient when the opportunity cost of funds is low. There may be multiple equilibria.

The paper is organized as follows. The model is presented in Section 1. The case of no verification cost (and perfect information *ex ante*) is analyzed in Section 2. Because of the search externality, there may be two steady state equilibria. In the first, loans are not rolled over when the investment project does not pay quickly. The

1. A complete account of the Japanese banking crisis can be found in Kanaya and Woo (2000).

2. For an analysis of multiple lenders' contracts with bankruptcy, see Bisin and Rampini (2006).

3. Diamond (1990) analyzes a model of pairwise credit in search equilibrium. He focuses on the externalities in trades. There is no distinction between banks and entrepreneurs and all agents may be lenders or borrowers when one party in the match cannot deliver the good for immediate exchange. There is also no distinction between different project types and hence no asymmetry of information between the parties of the match.

mass of banks and entrepreneurs searching is large and the probability of finding a match relatively high. Hence, the continuation of a project with slow return is less profitable than breaking the match to search for a new one. In the second equilibrium, loans are rolled over, more agents are tied in a relation and the probability of finding a match is smaller. The low return of the search compared to the return of the project continuation sustains an equilibrium that is socially inferior to the first one.

In Section 3, there is a fixed cost of verification on the lender and commitment: by assumption, there is no possibility of renegotiation after verification. The standard debt contract is shown to be privately optimal. The verification cost enhances the private incentives toward the continuation of a long-term project because of two effects. First, it lowers the profitability of the search for new opportunities as compared to rolling over a loan in the continuation of a match for which the verification cost has been paid. Second, the incentive to reduce the verification cost leads to the transfer of all payoffs to the lender after verification with an additional motive to pursue a match between the lender and the borrower in a long-term project that is privately but not socially efficient. Because of these effects, the socially efficient steady state without rollover may no longer be an equilibrium when verification is costly.

Section 4 presents a complete analysis of the dynamics for a model that is simplified: the lender is assumed to capture all the surplus of the match. The dynamics depend only on the stock of loans and the difference between the values of loans and loanable funds. Under suitable parameter conditions, we find again the two steady state equilibria with and without rollover with a unique path that converges to the steady state with perpetual rollover. But the set of equilibria is much richer than these two cases: all the other equilibria form a continuum of paths with cycles.

These cycles do not run smoothly. In each of them, the total amount of loans gradually increases until a crisis takes place. In a crisis, there is a quantum reduction of the stock of loans. On such a path, the value of the loan must eventually be reduced to that of loanable funds. At that instant, there must be a sudden reduction of the stock of loans. Unless all loans are terminated and the economy is in the steady state with no rollover, another such crisis must take place later. The cycles are not generated by the presence of a gradually overvalued collateral as in Kiyotaki and Moore (1997). We do not want to minimize the importance of such collateral effects. We only want to focus on another cause of credit cycles. Old projects with a low productivity are terminated and after the crisis, the productivity of the economy increases. Crises have a cleansing effect (Caballero and Hammour 1994).

This work is related to the issue of debt overhang[4] (Myers 1977). We show that lending for too long is socially inefficient. In particular, in the dynamics of the model, low-productivity loans are shown to be associated with lower levels of output. Low-productivity loans that are not terminated lead to underinvestment in good projects.

4. There is now a rich literature on the debt overhang. Among other studies, a large corporate debt reduces the return of new investment because of loan seniority in Lamont (1995). Snyder (1998) studies the debt overhang problem in the context of an entrepreneurial project requiring a sequence of investments financed by an outside lender. In his model, loan commitments with a fixed payment are optimal and dominate standard debt contracts because the interest of the debt contract has an adverse effect on the effort of the entrepreneur and on the probability of success of the project. Such an effect does not appear here because the probability of success is exogenous.

The current crisis has shown that nonperforming loans cannot be remedied by extra financing. In the same way, the extension of low-productivity loans through a rollover leads to a suboptimal outcome.

Labadie (1995) studies the impact of monetary policy on the level of financial intermediation. Lending is modeled in a way similar to this paper. She shows that when contracts can be written in a way that the real return to lending is unaffected by inflation, then real lending is unaffected by policy changes. When real returns are affected by inflation, lending increases in response to an expansionary monetary policy. In this paper, the extension of loans or their reduction is not a consequence of policy choices but rather the result of individual choices and opportunity costs that are endogenous to the general equilibrium.

1. THE MODEL

There are two types of agents, banks and entrepreneurs. Each type forms a continuum of a fixed mass, A for the entrepreneurs, and B for the banks. For simplicity and without loss of generality, we assume $A = B$. Each bank is endowed with a unit of resources that can be loaned to an entrepreneur. Entrepreneurs can undertake a project if they get a loan of one unit from a bank. The mass of loanable funds does not depend on previous profits in order to keep the model tractable and to avoid additional effects similar to those already analyzed in Bernanke and Gertler (1989). Entrepreneurs and banks meet through a matching process. Banks and entrepreneurs discount their payoffs at the same[5] rate, ρ.

1.1 Investment Projects

An essential feature of the model is that the return of an investment project is uncertain and not observable *ex ante*. Once the project is started, one may learn that its quality is lower than average and that the project requires more effort, resources and lending with a payoff to occur later than initially anticipated. For tractability, this feature is stylized here by assuming that there are only two types of projects, good or bad. The type of the project is not known *ex ante* to anyone, so there is no adverse selection. Once the project is undertaken, its type is known by its entrepreneur but it is known to the lender only if an observation is made at some cost. The probability of a good type is α which is publicly known.

In an actual economy, even good projects would take time to mature. In setting our model, we want to highlight the contrast of timing between good and bad projects and we want to keep it sufficiently simple for the dynamic analysis. For bad projects, the timing of a return is delayed and more uncertain. We assume that time is continuous and that the payoff of the good projects is immediate (after the loan is granted and the project has been implemented), while the bad projects generate a one time payoff according to a Poisson process. In this way, we avoid an heterogeneity of vintages

5. The results hold if banks have a strictly lower rate than entrepreneurs.

between bad projects of different ages that would make the model too complex for the dynamic analysis.

The instantaneous payoff of good projects is a random variable z that is distributed according to a density function $\phi(z)$ which is continuously differentiable and strictly positive on the interval $[0, C]$. The long-term project requires the extension of the bank loan. Such an action will be called rolling over the loan. For simplicity, we assume that the rollover does not involve any change with respect to the initial amount of the loan. The output of the long-term project is determined by a Poisson process that generates once the amount y, with probability λ per unit of time.[6]

1.2 The Matching Process

Let M be the mass of loans outstanding, which is also the mass of entrepreneurs on long-term projects. The mass of funds available for lending and the mass of entrepreneurs looking for a loan are equal to $A - M$. We assume that for a bank with a loanable fund, the probability of finding an entrepreneur is equal to μ, which is endogenous. That probability is also the probability of finding a bank for an entrepreneur who searches for a loanable fund in order to undertake a project. A standard specification in the literature (see Diamond 1982, 1990, Mortensen and Pissarides 1994, Rocheteau and Wright 2005) is that $\mu = \nu(A - M)$, where ν is a constant parameter. We only assume that μ is an increasing function[7] of $A - M$. Each match is a one time match between a bank and an entrepreneur as in Diamond (1990).

1.3 Imperfect Information

Neither a bank nor an entrepreneur knows the type of the project before it is started. The fraction of short-term projects, α, is known by all agents. After the investment is made, the entrepreneur observes the type of his project and its output if it is short term. Following the literature, it is assumed that banks observe the type of the project and its output only if they pay a cost that is assumed here to be fixed and equal to κ. The observation requires an examination of the books of the entrepreneur and is equivalent to a bankruptcy. In an actual bankruptcy, equity holders receive nothing and all that can be paid goes to the bank. We will show that this outcome is generated in the present setting by the optimal contract under imperfect information, as in the one-period model of Townsend (1979) and Gale-Hellwig (1985).

1.4 Bargaining for the Terms of the Loans

When a bank and an entrepreneur meet, they write a loan contract that generates a payoff S_B for the borrower (the entrepreneur) and S_L for the lender (the bank). It

6. The technology of long-term projects here is similar to the one in Hellwig (1977), but we ignore issues related to the dynamic structure of creditor–debtor interaction that led to ambiguity in the optimal creditor behavior about the decision to terminate a loan.

7. One could also assume that μ is constant and that there is a positive externality between the projects such that the probability that a particular project pays off quickly (i.e., is short-term) increases with the mass of the better projects in the economy. The fraction μ of new loans over available funds could also increase with the level of available funds if banks vary their loan criteria as in Rajan (1994), Asea and Blomberg (1998), and Dell'Ariccia and Marquez (2006).

is assumed that the contract maximizes under imperfect information, the function $\mathcal{U}(S_B, S_L)$. The function $\mathcal{U}$ is differentiable, strictly quasi-concave such that the marginal rate of substitution $\mathcal{U}_1/\mathcal{U}_2$ tends to ∞ if S_B/S_L tends to 0, and $\mathcal{U}_2/\mathcal{U}_1$ tends to ∞ if S_L/S_B tends to 0. The Nash bargaining solution corresponds to the special case of a function $\mathcal{U}$ with unit elasticity of substitution. The bargaining solution depends only on the indifference curves and not on the cardinal properties of $\mathcal{U}$, but the constant-returns-to-scale in the payoffs will provide an index to compare different equilibria.

2. PERFECT INFORMATION

In this section, we consider the case of perfect information as it will serve as a benchmark for the study of imperfect information and the general dynamics. Perfect information means that there is no cost of observation of the output of the investment project: $\kappa = 0$. In an established match, the decision to continue a long-term project with a loan rollover or to terminate the project and let the bank and the entrepreneur search for a new match depends on the relative payoffs of the long-term project and of the search for new projects. A contract specifies the maximum length T of the match and the contingent payments to both parties.

The payments are made at the time the project produces its output. Each party values a contract according to the surplus that is generated compared to the alternative of continuing the search. Let U_B, U_L be the utilities of the borrower and of the lender in the state of searching. S_B, S_L are the surpluses of entering the contract for each party. Under perfect information, the output of the project is distributed costlessly and the sum of the surpluses, $S_L + S_B$, does not depend on this distribution. There are two possible outcomes for a project.

(i) If the project generates a short-term payoff (with probability α), both lenders and borrowers go back to search within a vanishingly short time. The expected total surplus in this case is equal to the expected value of the output z:

$$\zeta = \int z\phi(z)dz. \tag{1}$$

(ii) If the project generates a long-term payoff, while it is continued, it generates a flow of surplus equal to

$$s = \lambda y - \rho(U_L + U_B), \tag{2}$$

where the first term is the total return per unit of time of the Poisson process with parameter λ with payoff y, and the second term is the opportunity cost of keeping the project going instead of searching. At the instant after the project is revealed to be long term, the present value of keeping the project going until time T is

$$\int_0^T (\lambda y - \rho(U_L + U_B))e^{-(\rho+\lambda)t}\, dt,$$

which can be written as

$$R(T) = (\lambda y - \rho(U_L + U_B))\frac{1 - e^{-(\rho+\lambda)T}}{\rho + \lambda}. \tag{3}$$

The total surplus of the project is the sum of the expected short-term and long-term components.

$$S_B + S_L = \alpha\zeta + (1 - \alpha)R(T). \tag{4}$$

This equation defines the *surplus possibility frontier* (SPF) for a match in the space (S_B, S_L). Because there is no information cost, it is a line of slope -1 with an intercept that depends on whether long-term projects are continued or not. In a steady state, the sign of the surplus flow of long-term projects, s in equation (2), determines whether long-term projects are either discontinued immediately (when $s < 0$), or are continued until they yield eventually a payoff (when $s > 0$).

2.1 The Steady State Equilibrium without Rollover

If the rate of return of long-term projects, λy, is less than the opportunity cost of using the funds for search, $\rho(U_L + U_B)$, then no rollover takes place. All matches are instantly profitable or fail and agents are always searching. When no fund is tied to long-term projects and all agents are searching, the probability of a match is $\bar{\mu} = \mu(A)$. This is the highest possible value of μ.

The return from searching is equal to the instantaneous probability of a match, $\bar{\mu}$, multiplied by the surplus generated in a match with no rollover, $\alpha\zeta$. The sum of the utilities

$$\bar{U}_L + \bar{U}_B = \frac{\bar{\mu}\alpha\zeta}{\rho}, \qquad \text{with} \qquad \bar{\mu} = \mu(A). \tag{5}$$

The necessary and sufficient condition for the steady state without rollover to be an equilibrium is that the surplus flow of long-term projects be negative (i.e., $s < 0$) which is equivalent to

$$\lambda y < \bar{\mu}\alpha\zeta. \tag{6}$$

In this steady state, the equation (4) of the surplus possibility frontier takes the form

$$S_L + S_B = \alpha\zeta, \tag{7}$$

and the distribution of the surplus is determined in an optimal contract that maximizes $\mathcal{U}(S_L, S_B)$ on the SPF.

2.2. *The Steady State with Rollover*

If the surplus flow of long-term projects is positive (i.e., $s > 0$), any ongoing loan is rolled over until it yields an output. The flow of terminations is therefore equal to λM^* where M^* is the stock of loans for ongoing long-term projects in the steady state. That value is determined by the equality between inflows and outflows in the stock of rolled over loans:

$$(1-\alpha)(A-M^*)\mu^* = \lambda M^*, \qquad \text{with} \qquad \mu^* = \mu(A-M^*). \tag{8}$$

The left-hand side is equal to the inflow of new loans on long-term projects and is the product of the flow of new loans, $(A-M^*)\mu^*$, with the share of the long-term projects, $1-\alpha$. The right-hand side is the flow of long-term loans that are repaid when the project succeeds. Since $\mu(A-M)$ is decreasing in M, in the steady state with rollover,

$$\mu^* = \mu(A-M^*) < \mu(A) = \bar{\mu}. \tag{9}$$

In the steady state with rollover, with $T = \infty$ in $R(T)$, the equation (4) of the SPF is

$$S_B + S_L = \alpha\zeta + \frac{1-\alpha}{\lambda+\rho}\big(\lambda y - \rho(U_B + U_L)\big). \tag{10}$$

Since the return of search, $\rho(U_L + U_B)$, is equal to the probability of a match μ^* multiplied by the total surplus $S_L + S_B$, using (10), the condition $s \geq 0$ is equivalent to

$$\lambda y \geq \mu^* \alpha\zeta, \tag{11}$$

where μ^* is the matching probability in the rollover steady state, defined in (9). To summarize the previous discussion, we introduce a notation and a proposition.

DEFINITION 1 *(threshold values of y). Let $\bar{y}$ and $\underline{y}$ be defined by*

$$\begin{aligned} &\lambda\bar{y} = \bar{\mu}\alpha\zeta, \quad \textit{with} \quad \bar{\mu} = \mu(A) \qquad \textit{and} \quad \zeta = \int z\phi(z)dz, \\ &\lambda\underline{y} = \mu^*\alpha\zeta, \quad \textit{with} \quad \mu^* = \mu(M^*) \quad \textit{and} \quad M^* \textit{given by (8)}. \end{aligned} \tag{12}$$

In Definition 1, $\bar{y}$ is the threshold value of y for which the rate of return of long-term projects (i.e., $\lambda\bar{y}$) is equal to the return from searching when the likelihood of a match is at its highest level, $\bar{\mu}$. $\underline{y}$ is similarly defined. The previous discussion leads to the next result.

PROPOSITION 1. *Under perfect information,*

(i) the steady state without rollover is an equilibrium if $y \leq \bar{y} = \bar{\mu}\dfrac{\alpha\zeta}{\lambda}$,

(ii) the steady state with rollover is an equilibrium if $y \geq \underline{y} = \mu^* \dfrac{\alpha\zeta}{\lambda}$.

Proposition 1 says that the payoff of long term projects needs to be small enough for the existence of a steady state without rollover, while it needs to be large enough for a steady state with rollover to obtain. In the range $y \in (\underline{y}, \bar{y})$, there are two steady state equilibria. The steady state with no rollover is superior as shown in the next result, which is proven in the Appendix.

PROPOSITION 2. *Assume that the parameters of the economy are such that under perfect information* ($\kappa = 0$)*, there are two equilibrium steady states and the economy is in the steady state with rollover. Then the termination of all loans induces a jump to the steady state without rollover. The steady state with rollover is strictly Pareto dominated by a jump to the steady state without rollover.*

Proposition 2 says that keeping the loans going until they return a payoff may be socially inefficient. Indeed, a termination of the loans would increase the utilities of all agents. The utilities of agents in the steady state with rollover is strictly lower because of the search externality. Fewer agents are searching because more agents are in an established match where a loan sustains the continuation of a long-term project. Such a long-term project is privately efficient in that equilibrium because of the lower opportunity cost of funds and searching. Banks lend for too long: loans for long-term projects should be called back. Such long-term projects would not be efficient in the equilibrium without rollover.

3. IMPERFECT INFORMATION WITH COMMITMENT

In the rest of the paper, the cost of observation by the lender, κ, is strictly positive. Once a contract is signed between a lender and a borrower, and before the undertaking of the project, there is no renegotiation if the project turns out to be long term. A contract with commitment is a binding agreement.

3.1 Efficient Contracts

A loan contract is defined by the maximum length of time, T, for the extension of the loan and the payments by the borrower to the lender before T.

In the present model, once the project has started, after a vanishingly small period of time, the entrepreneur knows whether the project has returned z or whether it will generate y according to a Poisson process. The lender does not have that information unless the costly monitoring takes place, but he knows that the entrepreneur is informed.

Following Townsend (1979) and Gale-Hellwig (1985), let b be the amount set in the contract such that if the entrepreneur pays $b \geq 0$, there is no verification at that instant (a vanishingly short time after the investment is made). If $b = 0$, then the entrepreneur can claim that the project has turned out to be of the short-term type with a return $z = 0$ and depart from the match. The lender would never receive anything. Hence, in an efficient contract, $b > 0$. But an entrepreneur can pay $b > 0$ only if the project has turned out to be short term with $z \geq b$. If the project turns out to be long term, the entrepreneur cannot meet the payment b and verification must take place. In that case, the lender learns that the project is long term. Given the structure of the model, there is no further need for verification later.

To summarize, in an efficient contract, no verification takes place if the borrower is able to pay b. In this case, the project's payoff is short term and equal to $z \geq b$: the borrower gets $z - b$ and goes back to search. Otherwise there is verification. The distribution of the returns between the lender and the borrower in this case is analyzed in the Appendix. Its property is similar to that in Townsend (1979) and Gale-Hellwig (1985): in order to save on monitoring cost, if a verification takes place, all investment's return goes to the lender in an efficient contract.[8]

PROPOSITION 3. *All efficient loan contracts are debt contracts* (b, T)*: if the borrower pays b there is no verification by the lender; if the borrower does not pay b, the lender pays the cost of verification and gets all the return from the project ; if the project is revealed to be long-term, the loan is extended until time* $T \in [0, \infty]$.

3.2 The Steady State Equilibrium with No Loan Rollover

We first consider a steady state where banks do not extend the time to repay the loan if the project has not succeeded in the short term. It will be shown that under some condition, such a policy is the result of an optimal contract between banks and entrepreneurs when they meet. When no loan is rolled over, the debt contract prescribes that if the borrower cannot pay b, the lender verifies, takes whatever the project has generated, and recalls the loan if the project turns out to be long term. The stock of loans M is therefore equal to zero and the masses of entrepreneurs and banks searching for a match are at their maximum, A. The probability of meeting a partner is also at its maximum, $\mu(A) = \bar{\mu}$.

The surplus possibility frontier and the equilibrium. In a steady state with no rollover, a match lasts a vanishingly small length of time. The surpluses of the borrower and the lender take into account the distribution of the project's return in the efficient contract of Proposition 3 and depend on the fixed payment b that is set in the contract. The entrepreneur-borrower gets a payoff only if the project's return is higher than the debt payment, b, and the expected value of that payoff (conditional on no verification) is $\omega(b) = \int_b (z - b)\phi(z)dz$. The surplus of the lender is written net of the cost of

8. Agents make decisions in a multiperiod environment, but the realization of the uncertainty and the asymmetric information between the borrower and the lender takes place within one period (which is assumed to be vanishingly short). Models of multiple periods debt contracts as in Chang (1990) are therefore not relevant.

verification:

$$S_B(b) = \alpha\omega(b), \text{ with } \omega(b) = \int_b (z-b)\phi(z)dz,$$
$$S_L(b) = \alpha\left(b\int_b \phi(z)dz + \int_0^b z\phi(z)dz\right) - \kappa\pi(b), \tag{13}$$

where $\pi(b)$ is the probability of verification which is the probability that the borrower does not meet the debt payment b:

$$\pi(b) = \alpha\int_0^b \phi(z)dz + 1 - \alpha. \tag{14}$$

Adding up the equations in (13), the total surplus to the borrower and the lender is

$$S_L(b) + S_L(b) = \alpha\zeta - \kappa\pi(b), \tag{15}$$

which is smaller than the surplus available under perfect information as expressed in (7). The difference is the expected cost of verification in the last term of (15). That cost increases with the debt payment b because of the higher probability of verification.

When the debt payment b varies, the point of coordinates $(S_B(b), S_L(b))$ moves on a curve that defines the frontier of the payoff possibility set. When b increases, the surplus of the borrower, $S_B(b)$, decreases. The lender's surplus increases by a smaller amount because of the additional cost of verification. Therefore, the slope of the surplus possibility frontier $\sigma(b) = S'_L(b)/S'_B(b)$ is strictly greater than –1:

$$\sigma(b) = -1 + \kappa\theta(b), \qquad \text{with} \qquad \theta(b) = \frac{\phi(b)}{\int_b \phi(z)dz} > 0. \tag{16}$$

The value of the debt payment b is chosen by the lender and the borrower to maximize the utility function $\mathcal{U}(S_B, S_L)$ on the payoff possibility set. The optimization problem has always a solution because the payoff set is bounded and $\mathcal{U}$ is continuous. In the steady state, agents receive a surplus with a probability $\bar{\mu}$ per unit of time. The utilities of the borrower and of the lender are the discounted present value of that stream:

$$U_B(b) = \frac{\bar{\mu}}{\rho}S_B(b) = \frac{\bar{\mu}}{\rho}\alpha\omega(b),$$
$$U_L(b) = \frac{\bar{\mu}}{\rho}S_L(b) = \frac{\bar{\mu}}{\rho}\Big(\alpha(\zeta - \omega(b)) - \kappa\pi(b)\Big). \tag{17}$$

Conditions for the no rollover equilibrium. The terms of the loans are (b, T), the debt payment and the maximum length of time for rolling over the loan. In the steady state with no rollover, $T = 0$. The optimality of no rollover is established by showing that extending the length of the contract, T, is suboptimal. Consider an increase of T by dT small. The borrower receives nothing as long as the contract is in place and foregoes the return from search. His opportunity cost is ρU_B. If the contract is extended by dT, his surplus decreases by $\rho U_B dT$, conditional on that extension being effective (i.e., the project turns out to be long-term and does not pay off before time T).

Likewise, the impact of the extension dT on the surplus of the lender, conditional on that extension being effective, is $(\lambda y - \rho U_L)dT$, where λdT is the probability of a payoff (that goes to the lender according to the contract), during the time interval dT.

The ratio of the two impacts is independent of T. Hence, in the diagram (S_B, S_L), when the length of the contract T increases from 0 to ∞, the point that represents (S_B, S_L) varies on a line segment SK in Figure 1 with slope $\eta(b)$ with

$$\eta(b) = -\frac{\lambda y - \rho U_L(b)}{\rho U_B(b)}. \tag{18}$$

That slope is negative for the case represented in the figure, where $\lambda y - \rho U_L(b) > 0$: there is a trade-off between the surpluses of the borrower and the lender: an extension of the length T always reduces the surplus of the borrower because he gets nothing after verification in the debt contract and foregoes the benefit of searching while he is tied in the match; the extension benefits the lender when the gross return from the project λy is greater than his opportunity cost $\rho U_L(b)$.

In Figure 1, values of $T > 0$ are inefficient. When the return of the long-term project is not too high, a contract (b, T) is dominated by a contract $(b', 0)$ with no rollover and a higher debt payment $b' > b$: if the project turns out to be short term, the borrower is immediately freed to get back to search for a new opportunity. The lender gets nothing, but he is compensated by the higher debt payment b' in the outcome of a short-term project.

One verifies in Figure 1 that the necessary and sufficient condition for the efficiency of the no rollover contract $(b, 0)$ is that the slope of the segment SK is greater than the slope of the tangent to the surplus possibility frontier, $\eta(\bar{b}) > \sigma(\bar{b})$. That condition is necessary and sufficient because of the quasi-concavity of the function $\mathcal{U}(S_B, S_L)$. We have established the following result.

PROPOSITION 4. *The steady state without rollover is an equilibrium if and only if*

$$\begin{aligned} &y \le \tilde{y}(\bar{b}, \bar{\mu}; \kappa), \\ &\textit{with} \quad \tilde{y}(b, \mu; \kappa) = \frac{\mu}{\lambda}\Big(\alpha\zeta - \kappa\pi(b) - \alpha\kappa\theta(b)\int_b (z - b)\phi(z)dz\Big), \end{aligned} \tag{19}$$

where $\bar{b}$ is the debt payment and $\bar{\mu}$ is the probability of a match in the equilibrium without rollover, and $\theta(\cdot)$ has been defined in (16).

In the argument before Proposition 4, we have shown that a shift from a contract (b, T) with $T > 0$ to a contract $(b', 0)$ with $b' > b$ is efficient if the return to long-term

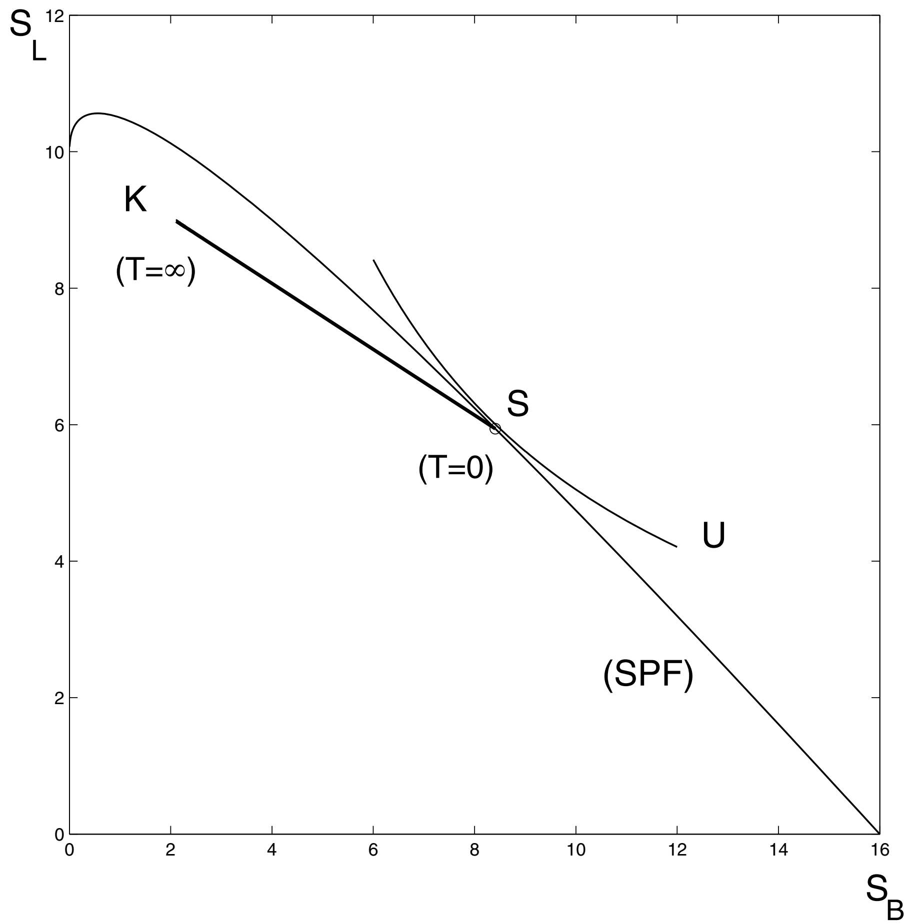

FIG. 1. Surpluses with Loan Rollover for Time T (in the steady state with no rollover).

NOTE: When loans are extended up to the maximum time T, the surplus allocation point moves on the segment SK.

projects is not too high. The higher debt payment $b' > b$ entails a higher verification cost. Proposition 4 shows that the higher verification cost reduces the upper bound $\tilde{y}$ of the range of values of y for which there is an equilibrium with no rollover as compared to the case of perfect information (Proposition 1).

When information is costly, the critical value $\bar{y}$ in Proposition 1 is reduced to $\tilde{y}(\bar{b}, \bar{\mu}, \kappa)$ in Proposition 4 because of the two cost terms in (19). The first, $\kappa\pi(b)$, is the average cost of verification *ex ante*. It would arise for any contract in the context of a costly transmission of information: the cost lowers the total *ex ante* surplus in any match (equation (15)), hence the value of using funds for searching instead of rolling over loans in current matches. The second term is specific to the debt contract.

It arises because of the particular incentive in that contract to reduce the information cost on the margin, and it depends on the density of the short-term return z near the debt payment b below which the information cost is paid.

3.3 The Steady State Equilibrium with Rollover

Assume now that the economy is in a steady state where lenders, after verification that the project is long term, roll over the loan until the project generates a payoff. The analysis proceeds along the same lines as in the previous case. The difference is that the probability of a match μ^*, in (9), is smaller than $\bar{\mu}$ because more agents are tied in loan relations (with a stock $M^* > 0$ given in (8)), and hence a larger proportion of agents are not searching.

Let b^* be the debt payment in the steady state (to be determined later). In a match the surplus for each agent is the sum of the surpluses generated by the short-term outcomes and the surpluses generated by the extension of the loan contract for long-term projects. The second component is positive for the lender as he gets all the payment in the debt contract, and negative for the borrower because of the foregone search opportunities. Accordingly, these surpluses can be written:

$$\begin{aligned} S_B^*(b^*) &= S_B(b^*) - (1-\alpha)\frac{\rho U_B^*}{\rho+\lambda}, \\ S_L^*(b^*) &= S_L(b^*) + (1-\alpha)\frac{\lambda y - \rho U_L^*}{\rho+\lambda}, \end{aligned} \tag{20}$$

where $S_B(b)$ and $S_L(b)$ are the surplus functions in a steady state with no rollover and debt payment b, as expressed in (13). The point $(S_B^*(b^*), S_L{}^*(b^*))$ is represented in Figure 2 by the point S^*. When the contract length is reduced from ∞, the surplus point moves on the segment S^*K^*, by the same argument as in the previous section: the lender gets less from the long-term project while the borrower gains from going back to search earlier. At the point K^*, there is no rollover. That point is therefore on the surplus possibility frontier[9] for no rollover contracts. That frontier was analyzed in the previous section and is represented here by the dashed curve in Figure 2, which also represents the segment SK of the previous section.

The slope of the segment S^*K^* is lower than that of the segment SK because in general equilibrium, $\mu^* < \bar{\mu}$ and the opportunity cost of search is lower for both the borrower and the lender. Following the argument of the previous section, an extension of the loan contract from T to $T + dT$ reduces the surplus of the borrower and increases that of the lender, but the reduction per unit of time, ρU_B^*, is now smaller while the increase, $\lambda y - \rho U_L^*$, is larger. The ratio of the second expression over the first is the absolute value of the slope of the segment S^*K^* which is larger than that of SK. By the same argument as for Proposition 4, the contract with rollover is efficient

9. The surplus possibility frontier is higher with rollover but since the probability of a match is lower, the expected discounted utilities of future matches is lower.

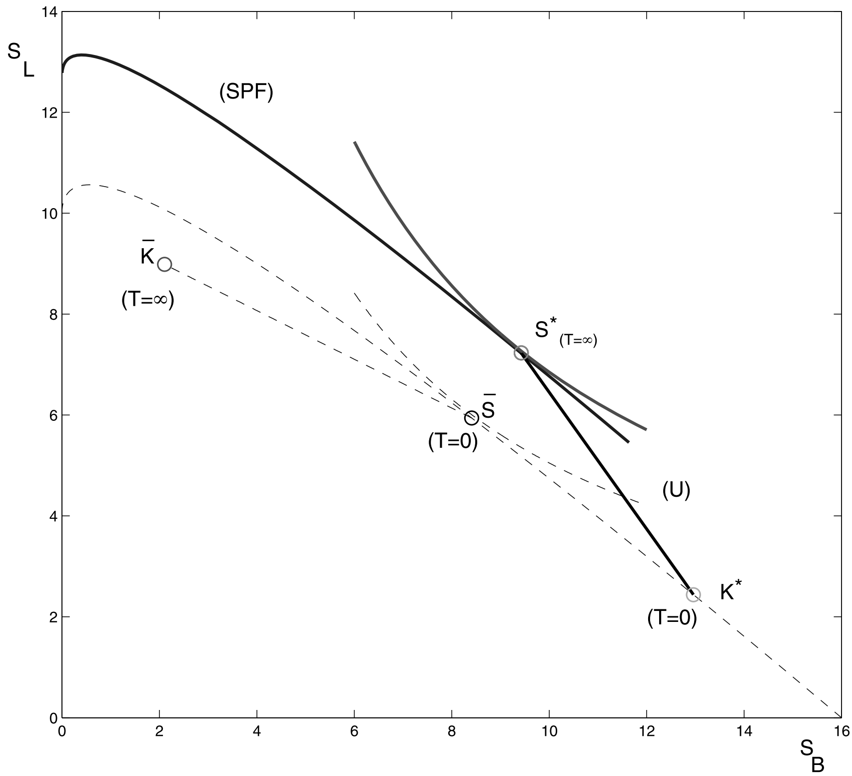

FIG. 2. Surpluses with Loan Rollover for Time T (in the steady state with rollover).

NOTE: The curves for the steady state with no rollover are in dotted lines where the points S and K in Figure 1 are now $\bar{S}$ and $\bar{K}$. For the equilibrium with perpetual rollover, the probability of a match, μ, is chosen for the sake of the figure at half the value of the steady state with no rollover (since there is *a priori* no restriction on the function $\mu(A - M)$). The equilibrium in the steady state with perpetual rollover is the point S^*. When loans are extended up to the maximum time T, the surplus allocation moves on the segment S^*K^* with $T = \infty$ at the point S^*.

when the slope of S^*K^* is smaller (algebraically) than the slope of the tangent to the surplus possibility frontier at the point S^*. In the equilibrium with rollover, the output y of the long-term project must be sufficiently *high* with respect to the opportunity cost, but it is not as high as in the perfect information case because of the cost of information. Imperfect information leads to a larger proportion of loans for long-term projects that are not called back. The condition for the equilibrium is the symmetric of Proposition 4 that is proven using (20) and standard algebra.

PROPOSITION 5. *The steady state with rollover is an equilibrium if and only if*

$$y \geq \tilde{y}(b^*, \mu^*; \kappa),$$

where the function $\tilde{y}(b^*, \mu^*; \kappa)$ is given in (19) of Proposition 4, and (b^*, μ^*) are the debt payment and the probability of a match in the equilibrium with rollover.

3.4 Comparing the Steady State Equilibria

The next statement extends Proposition 2 to the case of verification cost.

PROPOSITION 6. *For some strictly positive value of the cost of observation by the lender, $\bar{\kappa}$, if $\kappa < \bar{\kappa}$, Proposition 2 holds: if the economy is in the steady state equilibrium with rollover, the termination of all loans induces a jump to the steady state equilibrium with no rollover and an increase in the utility of all agents.*

The result follows immediately from a continuity argument using Proposition 2. Proposition 6 shows that the continuation of loans tied to long-term projects is socially inefficient. Note that the condition on the verification cost $\kappa < \bar{\kappa}$ is a sufficiency condition. The statement does not imply that the result is invalid for a large cost κ. The analysis of the dynamics of a simplified model with given surplus for the borrower in Section 4 will show a much stronger result: there is a continuum of equilibrium paths and the equilibrium steady state with no rollover generates a level of output that is higher than the output on any other equilibrium path at any time (Proposition 9).

4. DYNAMICS IN A SIMPLIFIED MODEL

In order to analyze the global dynamics, the model is simplified. The structure of investment projects and information is the same, but entrepreneurs have to put an effort with a fixed cost c, after they get a loan and before the start of the project. Furthermore, we assume that the entrepreneur's surplus of a match is negligible either because the lender has sufficient power to extract all the surplus or because the entrepreneur competes with at least one other entrepreneur when he applies for a bank loan.

According to the debt contract, an entrepreneur gets a payoff only when the project turns out a short-term return and its return z is greater than b. From the previous assumptions, the gross payoff of the entrepreneur is equal to his input cost. Hence, the value of b is determined by

$$c = \int_b (z - b)\phi(z)dz. \tag{21}$$

The expected payoff of the lender in a short-term project is equal to

$$x = \alpha\left(b\int_b \phi(z)dz + \int^b z\phi(z)dz \right) - \kappa\pi(b),$$

where b is solution of (21). In the right-hand side, the first term is the expected return that the lender gets when the project pays off in the short term and $\pi(b)$ is the probability of verification as defined in (14).

The lender can be in one of two states: searching for a loan or in a match while rolling over a loan. Let U_t be the utility of the lender while searching and W_t his utility in a match with a loan. U_t and W_t will also be called the values of loanable funds and loans. Because all long-term projects generate a payoff through the same Poisson process with a unique parameter λ, the value of a loan does not depend on its age. The choice between rolling over a loan or termination is made by the lender, and it depends only on the difference $V_t = W_t - U_t$ between the values of the loan and loanable funds. If $W_t - U_t < 0$, the bank strictly increases its utility by terminating the loan and going back to search.

If $V_t = W_t - U_t > 0$, the loan is rolled over and values of U_t and W_t satisfy the standard price equations

$$\begin{cases} \dot{U}_t = \rho U_t - \mu_t(x + (1-\alpha)(W_t - U_t)), \\ \dot{W}_t = \rho W_t - \lambda(y + U_t - W_t). \end{cases} \tag{22}$$

In the first equation, a loanable fund finds a match with probability μ_t (which varies inversely with the stock of loans over time), in which case, the lender gets the sum of x and, with probability $1-\alpha$, the surplus value of the loan over the loanable fund, $W_t - U_t$. The term ρU_t appears because of the discounting[10]. In the second equation, the loan generates with instantaneous probability λ a payoff y, in which case, it becomes a loanable fund and its value changes by $U_t - W_t$.

If $V_t > 0$, no loan is terminated unless its associated project delivers an output. By taking the difference between the two equations in (22),

$$\dot{V}_t = (\rho + \lambda + \mu_t(1-\alpha))V_t + \mu_t x - \lambda y.$$

In this expression, μ_t is an increasing function of the mass of searching agents, hence a decreasing function of the stock of loans M_t.

The variation of the stock of loans M_t is determined by the difference between the inflow of new loans and the outflow due to successful projects:

$$\dot{M}_t = \mu(M_t)(1-\alpha)(A - M_t) - \lambda M_t,$$

with $\mu(M)$ a decreasing function.

The economy is characterized by the variables (M_t, V_t). Their evolution in the interior of the first quadrant satisfies the sytem of dynamic equations

$$\begin{cases} \dot{M}_t = \mu(M_t)(1-\alpha)(A - M_t) - \lambda M_t, \\ \dot{V}_t = (\rho + \lambda + \mu(1-\alpha))V_t + \mu(M_t)x - \lambda y. \end{cases} \tag{23}$$

10. If an asset generates a flow of income u_t, its price is $U_t = \int_t^{\infty} e^{\rho(t-s)} u_s ds$. By differentiation, $\dot{U}_t = \rho U_t - u_t$.

The evolution of the variables on the frontier of the first quadrant will be analyzed separately. Unless specified otherwise, it is assumed that all agents have perfect foresight on the dynamics of the economy.

4.1 Steady States

The steady state with rollover. The dynamic system (23) has a unique steady state that is characterized by the equation

$$1 - \alpha = \frac{\lambda M^*}{\mu(M^*)(A - M^*)}. \tag{24}$$

Since $\mu(\,\cdot\,)$ is decreasing, the first equation has a unique solution M^*. Let $\mu^* = \mu(M^*)$. Equation (24) defines the steady state with rollover. For that steady state to be an equilibrium the value of loans must be at least equal to that of loanable funds. The difference, V, is given by the stationary form of the dynamics of V_t in equation (23):

$$V = f(M) = \frac{\lambda y - \mu(M)x}{\rho + \lambda + \mu(M)\big(1 - \alpha\big)}. \tag{25}$$

The function $f(M)$ is strictly increasing and the steady state with rollover exists if and only if $f(M^*) \geq 0$, that is if $\lambda y > \mu^* x$: the long-term projects must be sufficiently productive as in Proposition 5.

The steady state with no rollover. In that steady state, the stock of loans is zero and the matching probability is $\bar{\mu}$. The value of loanable funds is $\bar{U} = \bar{\mu}x/\rho$. The steady state is an equilibrium if and only if the value of loans is less than the value of loanable funds ($f(M) \leq 0$ in (25)), which is equivalent to $\lambda y \leq \bar{\mu}x$. As in Proposition 4, the payoff of long-term project should not be too large. If $\lambda y < \bar{\mu}x$, any loan should be terminated immediately and turned into a loanable fund. The value of loans is therefore equal to the value of loanable funds and the difference between the two values, $\bar{V}$, is equal to 0. The next result is the equivalent of Propositions 4 and 5 and summarize the previous discussion.

PROPOSITION 7.

(i) There exists an equilibrium steady state without rollover if and only if $\lambda y \leq \bar{\mu}x$, with $\bar{\mu} = \mu(0)$. In that steady state,

$$\bar{U} = \frac{\bar{\mu}}{\rho}x. \tag{26}$$

(ii) There is an equilibrium steady state with perpetual rollover if and only if $\mu^* x \leq \lambda y$, with $\mu^* = \mu(M^*)$ and M^* solution in (24). In this case,

$$V^* = \frac{\lambda y - \mu^* x}{\rho + \lambda + (1 - \alpha)\mu^*}. \tag{27}$$

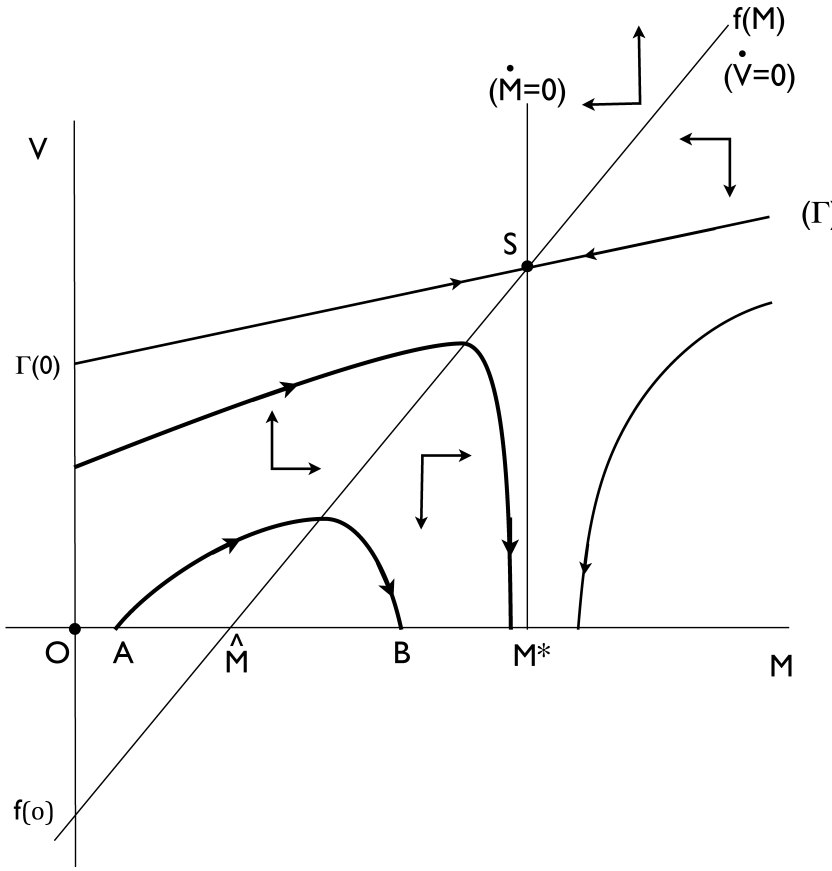

FIG. 3. Dynamics with Moderate Productivity of Long-Term Projects.

NOTE: The parameters are such that the two steady states exist and long-term projects are sufficiently productive to sustain a transition from a state with no loan to a state with perpetual rollover.

4.2 Equilibrium Dynamics with Multiple Steady States

The most interesting case is presented by parameters such that there are two steady state equilibria, without and with rollover, respectively. Following the previous proposition, we make the next assumption.

ASSUMPTION 1. $\mu^* < \dfrac{\lambda y}{x} < \bar{\mu}$.

The dynamics of the stock of loans, M_t, and of the difference between the values of loaned and loanable funds, V_t, as specified by the system (23) are represented in the space (M_t, V_t) of Figure 3. The steady state with rollover is represented by the point S that is saddle-point stable. There is a unique path (Γ) that converges to that steady state. The steady state with no rollover is represented by point O at the origin.

At that point, the stock of loans is equal to 0. The value of V is taken to be 0 by an abuse of notation.

A simple exercise shows that the graph of $f(M)$ which is the locus of $\dot{V} = 0$ intersects the horizontal axis at the point $\hat{M} \in (0, M^*)$ that is defined by

$$\mu(\hat{M}) = \frac{\lambda y}{x}. \tag{28}$$

We now distinguish two subcases that depend on the productivity of long-term projects relative to short-term projects as measured by $\lambda y/x$. In the first case, this productivity is high and $\lambda y/x$ is near the upper bound of the interval $(\mu^*, \bar{\mu})$. In the second case, $\lambda y/x$ is closer to the lower bound of the interval.

Case I: High productivity of long-term projects. When $\lambda y/x$ is sufficiently close to $\bar{\mu}$, the value of $|f(0)|$, as expressed in (25), is small and the path (Γ), which is above the graph of f, is such that $\Gamma(0) > 0$. This is the case that is represented in Figure 3. The stock of loans at time 0 is assumed to be smaller than the steady state value M^*: $M_0 < M^*$. The case $M_0 > M^*$ is similar.

The equilibrium with perpetual rollover. The unique path (Γ in Figure 3) that converges to the steady state is indeed an equilibrium. For any value of the initial amount of loans, M_0, there exists a unique value V_0 such that the economy converges to the steady state S in a perfect foresight equilibrium.

At each instant on the equilibrium path, the value of a unit of funds in a continuing loan is strictly greater than that of a liquid fund ready to be loaned: $V = W - U > 0$. The financial sector rolls over any existing loan that has not yet been paid off. On this path, the stock of loans on bad projects increases monotonically. The creation of new projects which is proportional to the stock of liquid assets, $A - M$, decreases monotonically.

The path with a unique crisis. Assume now that for any $M_0 \in [0, M^*)$, the initial value $V_0 = W_0 - U_0$ is strictly positive and smaller than the value $\Gamma(M_0)$. The economy is set on a dynamic path where the value of V is nil at some finite date (see Figure 3), T. At that date, the continuation of the dynamics along the path determined by (23) is no longer an equilibrium: the value of a unit of fund rolled over would be strictly smaller than that if it were recalled and used as liquidity for new loans. A "crisis" takes place. The value of V cannot jump because of the perfect foresight of agents. The only possible outcome is a sudden cut in the aggregate amount of loans to the reduced level M_T^+ belonging to the interval $[0, \hat{M}]$.

The value of postcrisis M_T^+ is not indeterminate. It depends on the values of loanable funds U_t and loaned funds W_t before the crisis. Consider the particular case of a crisis where all loans are terminated ($M_T^+ = 0$), and the equilibrium after time T is the steady state with no rollover as described above. In that steady state, the value of loanable funds is equal to $\bar{U}$ defined by equation (26) in Proposition 7. Hence, by time continuity of the value function U_t under perfect foresight, and because $V_T = 0$,

$$U_T = W_T = \bar{U}.$$

The values of U_t and W_t on the time interval [0, T] are determined by backward integration from time T to time t of the differential equations obtained from (22) and (23).

$$\begin{cases} \dot{V}_t &= \big(\rho + \lambda + \mu(M_t)(1-\alpha)\big)V_t + \mu x - \lambda y, \\ \dot{U}_t &= \rho U_t - \mu(M_t)\big(x + (1-\alpha)V_t\big), \\ \dot{W}_t &= \rho W_t + \lambda(V_t - y), \end{cases} \tag{29}$$

with the initial conditions $V_T = 0, U_T = W_T = \bar{U}$. For the given stock of loans M_0 and a time T, there is a unique value (U_0, W_0) such that on the dynamic path, no loan is terminated before time T when a crisis occurs and all loans are terminated from that time on. Such a path is illustrated in Figure 3 for $M_0 = 0$.

Paths with repeated crises or cycles. Assume now that at the time T of the crisis, not all loans are terminated and that the stock of loans just after time T is M_T^+ with

$$0 < M_T^+ < \hat{M}. \tag{30}$$

The economy just after time T is represented by the point A in Figure 3. After time T, the economy sets on a path that is determined by the dynamic equations (23). That path reaches the horizontal axis at some time T', at the point B with an amount of loans $M_{T'}$. During the interval of time (T, T'), the value of loans is strictly higher than that of loanable funds, with a difference V_t that first increases and then decreases to reach 0 at time T'. In that regime, no loan is terminated for a long-term project that has not delivered yet. A new crisis occurs at time T'. The amount of termination at time T', and the amount of surviving loans postcrisis, $M_{T'}^+$ depend on the values $U_{T'}$ and $W_{T'}$ just before the crisis, which depend on their initial values at time 0, as we have seen in the previous case of a dynamic path with a unique crisis.

Any path that satisfies the dynamic system (23) when $V > 0$ and has a quantum reduction of loans when V reaches 0 defines an equilibrium, provided that the loan amount after a crisis, M^+ satisfies the equation

$$M^+ < \hat{M}, \tag{31}$$

where $\hat{M}$ is defined by (28) (see Figure 3).

In general, an initial value (M_0, U_0, W_0) such that $W_0 - U_0 < \Gamma(M_0)$ generates an equilibrium path with at least one crisis. What happens after the crisis depends on the initial conditions. Except for a set of measure zero on (M_0, U_0, W_0), the economy goes through endless cycles that are marked by crises with loan reductions. Each regime between two consecutive crises is represented by an arc in Figure 3. All these arcs must be below the arc that starts at the point O with the longest time between two consecutive crises.

It is intuitive and illustrated in Figure 3 that the smaller the amount of loan reduction at the time of crisis, the higher the remaining stock $M^+ < \hat{M}$ and the shorter the time length of the next cycle. At the limit, if M^+ is taken arbitrarily close to $\hat{M}$, the economy is in an equilibrium steady state with an outflow of loans associated to long-term projects that has not delivered yet. The analysis of that state is left to the reader.

If $M_0 > M^*$, one can easily verify in Figure 3 that (i) there is a unique path that converges to S, on which no loan is terminated; (ii) all other paths (which depend on U_0 and W_0) generate a crisis in finite time after which the stock of loans is reduced to a value strictly below $\hat{M}$ and the previous discussion applies.

Case II: Low productivity of long-term projects. The difference between the values of loans and loanable funds in the steady state with rollover, given in equation (27) of Proposition 7, depends positively on the productivity of long-term projects, λy. For a sufficiently low productivity, the unique path $\Gamma(M)$ that converges to the steady state is such that $\Gamma(0)$ is strictly negative. This case is represented in Figure 4.

Let N be the amount of loans such that $\Gamma(N) = 0$. The point $(N, 0)$ is represented in Figure 4 by the same notation N. If at time 0, the stock of loans M_0 is smaller than N, then the only equilibrium is a termination of all loans after which the economy stays in the steady state with no rollover (point O in the figure). For $M_0 \geq N$, the analysis is the same as in the previous case. Note that the postcrisis stock of loans, M^+ must be such that $M^+ \geq N$, or $M^+ = 0$. In the second case, the economy is set in the steady state O.

4.3 Output and Efficiency

In the simplified model of this section, competition between entrepreneurs drives their surplus to zero and the total income of banks is equal to the aggregate income in the economy. The level of output is a function of the stock of loans, M:

$$Y(M) = \mu(M)x(1 - M) + \lambda y M, \tag{32}$$

where the matching probability for new loans $\mu(M)$, is a decreasing function of M. Total output is the sum of the outputs generated by the projects that perform rapidly and by the protracted projects with loans that are rolled over. Taking the derivative (which is assumed to exist),

$$Y'(M) = \mu'(M)x(1 - M) + \lambda y - \mu(M)x.$$

When the stock of loans M increases, there are two effects. The first is a reduction of the matching probability μ which reduces the flow of new projects and has a negative impact on output. The second is generated by the difference between the return of funds used in protracted projects, λy, and the return of funds employed in

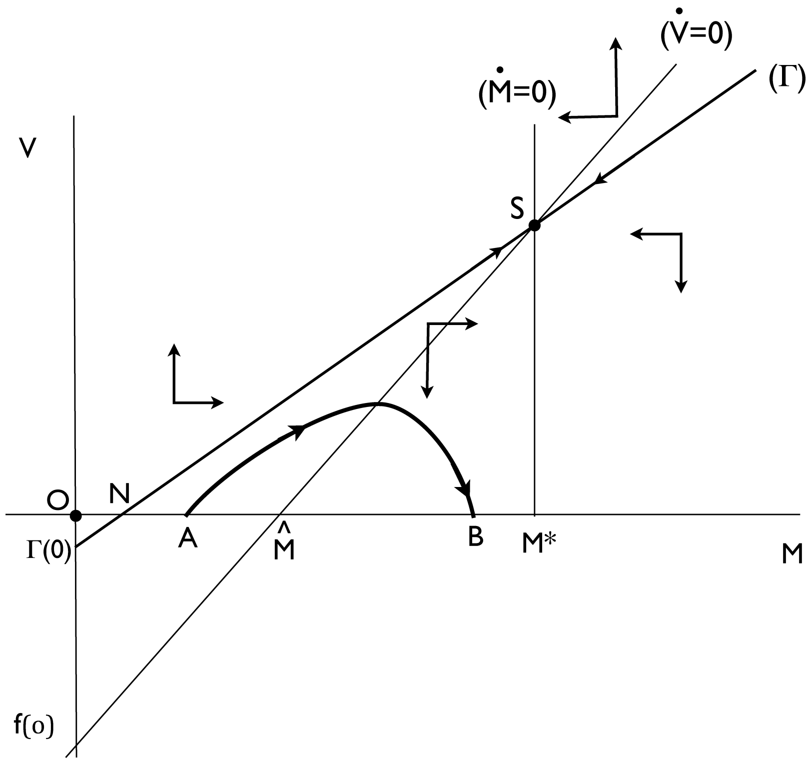

FIG. 4. Dynamics with Low Productivity of Long-Term Projects.

the search for good projects with a quick payoff, μx. By definition of $\hat{M}$ in (28), $\lambda y - \mu(M)x < 0$ if and only if $M < \hat{M}$. In that case, the overall impact of M on output is unambiguously negative.

PROPOSITION 8. *If $M < \hat{M}$ defined in (28), the level of output is a decreasing function of the stock of loans M.*

The result implies that for any $M \leq \hat{M}$, output $Y(M)$ is lower than in the steady state with no rollover, $Y(0) = \bar{Y}$.

When the stock of loans is sufficiently larger than $\hat{M}$ and μ is sufficiently low, funds may be more productive in supporting protracted projects than searching for new projects, and the marginal impact of a higher M on output may be positive. Assume for example that $\mu = \gamma(1 - M)^{\beta}$, with $\beta > 0$:

$$Y'(M) = \lambda y - (1 + \beta)\mu(M)x.$$

One can find parameters of the model such that $\lambda y > (1 + \beta)\mu(M^*)x$. Note that such an inequality is stronger than the condition of the left inequality in Assumption 1

($\lambda y > \mu(M^*)x$). In that case, if M is sufficiently close to M^*, output is an increasing function of M.

Although the marginal impact of M on output may be positive if M is sufficiently large, the next result (proven in the Appendix) shows that for any $M > 0$, the level of output is still strictly smaller than in the no-rollover steady state, $Y(0) = \bar{Y}$.

PROPOSITION 9. *Under Assumption 1, at any time where the stock of loans M_t is in the interval $(0, M^*)$, the level of output Y_t is strictly smaller than in the steady state with no rollover.*

The result is stronger than Proposition 6 that compared only the two steady states in the model of the previous section. From the two previous results, a crisis with its sudden reduction of loans has a unambiguous and positive effect on output only if the pre-crisis stock of loan is not too large compared to $\hat{M}$. In any case, peaks of output on equilibrium paths with crises are always below the level in the steady state with no rollover.

5. CONCLUSION

The opportunity cost of funds is instrumental in deciding whether to roll over or to call a loan back. When there are many opportunities for the use of funds in new projects, it may be better in a privately efficient contract between a lender and a borrower to terminate projects that do not provide an early payoff. However, the opportunities for new projects may be lower when borrowers and lenders are in a relation with loan rollover. The loan rollover may be privately efficient when the opportunity cost of funds is low. Lenders then have strong private incentive to keep the loan–investment relation to recover the payoff, which leads to an equilibrium where banks lend for too long.

We are aware that the termination of loans and investment projects may generate some social inefficiencies and the paper does not attempt to provide a general welfare analysis of crises with loan terminations. The results highlight the potential benefits that can arise from the cleansing effect of these crises: when old loans and projects are retained because the structure of the contract payments benefits the lenders, a large termination of loans would redirect loanable resources toward projects that are socially more efficient.

The paper could be extended in a number of directions. Whether lending is based on collateral or cashflow considerations influences lending standards and growth prospects along the economic cycle. In future work, we will extend the paper to study the role of collateral. We would also like to study the role of moral hazard incentives in the lending process, in view of the Japanese experience. Finally, the rolling over of loans generated here endogenous cycles without exogenous shock because of the search externalities. The next step, which may be more technical would be to analyze whether the loan rollovers that are induced by the debt contract amplify or extend cycles that are generated by exogenous shocks.

APPENDIX

PROOF OF PROPOSITION 2. In the steady state with rollover, from (3) and (4), the set of possible utilities for the two agents in the space (U_L, U_B) is determined by the equation

$$U_L + U_B \leq \frac{\mu^*}{\rho}\left(\alpha\zeta + (1-\alpha)\frac{\lambda y - \rho(U_L + U_B)}{\rho + \lambda}\right). \tag{A1}$$

In the steady state with no rollover, the set has the equation

$$U_L + U_B \leq \frac{\bar{\mu}}{\rho}\alpha\zeta. \tag{A2}$$

Using (6), the first frontier defined by (A1) is strictly inside the second one. A contract that maximizes the function $\mathcal{U}(S_L, S_B)$ under (A2) generates a utility for each agent that is stricly higher than under the constraint (A1). □

PROOF OF PROPOSITION 3. Let $h(z)$ be the payment to the lender if the output z occurs immediately after the project is implemented. Let δ be the payment to the lender when the project produces y according to the Poisson process. The contract specifies T, δ, and the function $h(z)$. The payoff of the loan to the lender is

$$\begin{aligned} S_L = {} & \alpha\left(b\int_b \phi(z)dz + \int^b \phi(z)(h(z) - \kappa)dz\right) \\ & + (1-\alpha)\Big(-\kappa + (U+\delta)\frac{\lambda}{\rho+\lambda}(1 - e^{-(\rho+\lambda)T}) + e^{-(\rho+\lambda)T}U\Big), \end{aligned}$$

where U is the utility of the lender when searching. Let u be the utility of the entrepreneur when searching. His payoff in the contract is

$$\begin{aligned} S_B = {} & \alpha\left(\int_b (z-b)\phi(z)dz + \int^b \phi(z)(z - h(z))dz\right) \\ & +(1-\alpha)\left((y - \delta + u)\frac{\lambda}{\rho' + \lambda}(1 - e^{-(\rho'+\lambda)T}) + e^{-(\rho'+\lambda)T}u\right). \end{aligned}$$

The sum of the payoffs is

$$S_L + S_B = \alpha \int z\phi(z)dz - (1-\alpha)\kappa - \kappa\alpha \int^b \phi(z)dz - \delta G_1 + G_2,$$

where

$$G_1 = (1-\alpha)\lambda\left(\frac{1-e^{-(\rho+\lambda)T}}{\rho+\lambda} - \frac{1-e^{-(\rho+\lambda)T}}{\rho+\lambda}\right)$$

$$G_2 = (1-\alpha)\left(U\frac{\lambda}{\rho+\lambda}(1-e^{-(\rho+\lambda)T}) + e^{-(\rho+\lambda)T}U\right) + (1-\alpha)(u+y)\left(\frac{\lambda}{\rho+\lambda}(1-e^{-(\rho+\lambda)T}) + e^{-(\rho+\lambda)T}\right)$$

are independent of h and δ and $G_1 \geq 0$.

In an efficient contract, $h(z)$ and δ maximize the payoff to, say, the lender subject to a given payoff for the borrower, or equivalently the sum of the payoffs. Omitting constant terms, we need to find $h(z)$ and δ to maximize

$$\alpha\left(b\int_b \phi(z)dz + \int^b (h(z)-\kappa)\phi(z)dz\right) + \delta(1-\alpha)\frac{\lambda}{\rho+\lambda}(1-e^{-(\rho+\lambda)T})$$

subject to $h(z) \leq z$, $\delta \leq y$, and

$$\kappa\alpha\int^b \phi(z)dz + \delta G_1 \leq P,$$

for some constant P. It is immediate that the solution is $h(z) = z$ and $\delta = y$. □

Proof of Proposition 9. From Proposition 8, we only need to consider the case where $M > \hat{M}$ and therefore $\lambda y - \mu(M)x > 0$. From (23), on the path (Γ) with $M \leq M^*$ that converges to the steady state with rollover, as $\dot{M}_t > 0$,

$$M_t = \frac{(1-\alpha)\mu_t - \dot{M}_t}{\lambda + (1-\alpha)\mu_t} < \frac{(1-\alpha)\mu_t}{\lambda + (1-\alpha)\mu_t}.$$

Using the function Y of M in (32), and omitting for clarity the arguments or the time subscripts,

$$Y = \mu x + (\lambda y - \mu x)M < \mu x + (\lambda y - \mu x)\frac{(1-\alpha)\mu}{\lambda + (1-\alpha)\mu} < \bar{\mu}x = \bar{Y},$$

where the last inequality is obtained by straightforward algebra and using $\lambda y < \bar{\mu}x$ and $\mu < \bar{\mu}$.

At any time for any M, the level of output depends only on the stock of loans M and is therefore the same as at the point of the path (Γ) with the same M. □

LITERATURE CITED

Asea, Patrick K., and Brock Blomberg. (1998) "Lending Cycles." *Journal of Econometrics*, 83, 89–128.

Bernanke, Ben S., and Mark Gertler. (1989) "Agency Costs, Net Worth, and Business Fluctuations." *American Economic Review*, 79, 14–31.

Bisin, Alberto, and Adriano A. Rampini. (2006) "Exclusive Contracts and the Institution of Bankruptcy." *Economic Theory*, 27, 277–304.

Caballero, Ricardo, and Mohamad Hammour. (1994) "The Cleansing Effect of Recessions." *American Economic Review*, 84, 1350–68.

Chang, Chun. (1990) "The Dynamic Structure of Optimal Debt Contracts." *Journal of Economic Theory*, 52, 68–86.

Dell'Ariccia, Giovanni, and Robert Marquez. (2006) "Lending Booms and Lending Standards." *Journal of Finance*, 61, 2511–46.

Diamond, Peter A. (1982) "Aggregate Demand Management in Search Equilibrium." *Journal of Political Economy*, 90, 881–94.

Diamond, Peter A. (1990) "Pairwise Credit in Search Equilibrium." *Quarterly Journal of Economics*, 105, 285–319.

Gale, Douglas, and Martin Hellwig. (1985) "Incentive-Compatible Debt Contracts: The One-Period Problem." *Review of Economic Studies*, 52, 647–63.

Hellwig, Martin. (1977) "A Model of Borrowing and Lending with Bankrupcy." *Econometrica*, 45, 1879–1906.

Kanaya, Akihiro, and David Woo. (2000) "The Japanese Banking Crisis of the 1990s: Sources and Lessons." IMF Working Paper 00/7.

Kiyotaki, Nobuhiro. (1998) "Credit and Business Cycles." *Japanese Economic Review*, 49, 18–35.

Labadie, Pamela. (1995) "Financial Intermediation and Monetary Policy in a General Equilibrium Banking Model." *Journal of Money Credit and Banking*, 27, 1290–1315.

Lamont, Owen. (1995) "Corporate-Eebt Overhang and Macroeconomic Expectations." *American Economic Review*, 85, 1106–17.

Mortensen, Dale, and Christopher Pissarides. (1994) "Job Creation and Job Destruction in the Theory of Unemployment." *Review of Economic Studies*, 61, 397–415.

Myers, Stewart C. (1977) "Determinants of Corporate Borrowing." *Journal of Financial Economics*, 5, 147–75.

Rajan, Raghuram. (1994) "Why Bank Credit Policies Fluctuate: A Theory and Some Evidence." *Quarterly Journal of Economics*, 109, 399–441.

Rocheteau, Guillaume, and Randall Wright. (2005) "Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium." *Econometrica*, 73, 175–202.

Snyder, Christopher M. (1998) "Loan Commitments and the Debt Overhang Problem." *Journal of Financial and Quantitative Analysis*, 33, 87–116.

Townsend, Robert M. (1979) "Optimal Contracts and Competitive Markets with Costly State Verification." *Journal of Economic Theory*, 21, 265–93.